A self-consistent relativistic study of nuclear spin-isospin resonances

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2 Theoretical Framework
   - Relativistic Hartree-Fock theory
   - Random Phase Approximation
   - RHF+RPA

3 Spin-isospin Resonances

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Nuclear spin-isospin resonances

- Nuclear charge-exchange excitations
  - $\beta$-decay
  - charge-exchange reactions

- These excitations play important roles
  - spin and isospin properties of the in-medium nuclear interaction
  - $\beta$-decay rates of nuclei in r-process path \textit{Engel:1999, Borzov:2006}
  - inclusive neutrino-nucleus cross sections \textit{Kolbe:2003, Vogel:2006}
  - isospin corrections for superallowed $\beta$ decays \textit{Towner and Hardy:2010}
  - $\beta\beta$-decay rates \textit{Ejiri:2000, Avignone:2008}

- Nuclear spin-isospin resonances become one of the central topics in nuclear physics and astrophysics.
Microscopic theories for spin-isospin resonances

- Shell models ($A \sim 60$)
  

- Random Phase Approximation (RPA) based on density functional theories
  - traditional (non-relativistic) density functional
    
  - covariant (relativistic) density functional: RH+RPA, RHF+RPA
Covariant density functional theory – RH theory

- Covariant density functional theory in Hartree level (RH/RMF theory) has received wide attention due to its successful description of lots of nuclear phenomena.
    - spin-orbit splittings
    - nuclear saturation properties (the Coester line) Brockmann:1990,1992
    - binding energy per nucleon $E/A$ Reinhard:1989, Ring:1996
    - isotopic shifts in the Pb region Sharma:1993
    - spin symmetry in anti-nucleon spectrum Zhou:2003
    - .....
RH+RPA for spin-isospin resonances

- RH+RPA for spin-isospin resonances

Example: Gamow-Teller resonance (GTR) in $^{208}$Pb ($\Delta S = 1$, $\Delta L = 0$, $J^\pi = 1^+$)
RH$^+$RPA for spin-isospin resonances

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\[
\begin{align*}
\text{RH+RPA for spin-isospin resonances} \\
\text{example: Gamow-Teller resonance (GTR) in } ^{208}\text{Pb} \ (\Delta S = 1, \Delta L = 0, J^\pi = 1^+) \\
\end{align*}
\]
RH+RPA for spin-isospin resonances


example: Gamow-Teller resonance (GTR) in $^{208}$Pb ($\Delta S = 1, \Delta L = 0, J^{\pi} = 1^+$)

- RH+RPA for spin-isospin resonances

- add $\pi$-meson

- fit $g'$

- self-consistency is missing
Covariant density functional theory – RHF theory

- Covariant density functional theory in Hartree-Fock level (RHF theory) for nuclear ground-state properties
  - several attempts to include the Fock term in the relativistic framework
  - DDRHF theory achieved quantitative descriptions of binding energies and radii on the same level as RH theory
    Long, Giai, Meng, PLB 640, 150 (2006), Long, Sagawa, Giai, Meng, PRC 76, 034314 (2007)
  - improvement on the descriptions of the nuclear shell structures and their evolutions
    Long, Sagawa, Giai, Meng, PRC 76, 034314 (2007), Long, Sagawa, Meng, Giai, EPL 82, 12001 (2008)
  - isospin properties of nuclear matter and neutron stars at high densities
    Sun, Long, Meng, Lombardo, PRC 78, 065805 (2008)
  - spin and pseudospin symmetries in nucleon spectra
Can spin-isospin resonances and related weak interaction processes be described fully self-consistently in covariant density functional theory?

**In this work**
- Fully self-consistent RPA approach is established based on the RHF theory.
- Applications
  - nuclear spin-isospin resonances
  - isospin symmetry-breaking corrections for the superallowed $\beta$ decays
  - inclusive charged-current neutrino-nucleus cross sections
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Covariant density functional theory – RHF theory

- Effective Lagrangian density \( \text{Bouyssy:1987, Long:2006} \)

\[
\mathcal{L} = \bar{\psi} \left[ i \gamma^\mu \partial_\mu - M - g_\sigma \sigma - \gamma^\mu \left( g_\omega \omega_\mu + g_\rho \vec{\tau} \cdot \vec{\rho}_\mu + e \frac{1 - \tau_3}{2} A_\mu \right) - \frac{f_\pi}{m_\pi} \gamma^5 \gamma^\mu \partial_\mu \vec{\tau} \cdot \vec{\tau} \right] \psi \\
+ \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \vec{R}_{\mu\nu} \cdot \vec{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}_\mu \\
+ \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} - \frac{1}{2} \frac{m_\pi^2}{m_\pi} \vec{\pi} \cdot \vec{\pi} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \\
\] (1)

- Energy functional of the system

\[
E = \langle \Phi_0 | H | \Phi_0 \rangle = E_k + E_\sigma^D + E_\omega^D + E_\rho^D + E_A^D + E_\sigma^E + E_\omega^E + E_\rho^E + E_\pi^E + E_A^E \\
\] (2)
Random Phase Approximation

- **RPA equations**

  \[
  \begin{pmatrix}
  \mathcal{A} & \mathcal{B} \\
  -\mathcal{B} & -\mathcal{A}
  \end{pmatrix}
  \begin{pmatrix}
  X \\
  Y
  \end{pmatrix}
  = \omega \nu
  \begin{pmatrix}
  X \\
  Y
  \end{pmatrix}
  \tag{3}
  \]

  where the matrix elements of particle-hole residual interactions read

  \[
  \mathcal{A} = \begin{pmatrix}
  (E_A - E_a)\delta_{AB}\delta_{ab} \\
  (E_\alpha - E_a)\delta_{\alpha\beta}\delta_{ab}
  \end{pmatrix}
  \]

  \[
  \mathcal{B} = \begin{pmatrix}
  \langle f_A f_B | V | f_b f_a - f_a f_B \rangle \\
  \langle f_\alpha f_B | V | f_b f_a - f_a f_B \rangle
  \end{pmatrix}
  \begin{pmatrix}
  \langle f_A f_B | V | f_b f_a - f_a f_B \rangle \\
  \langle f_\alpha f_B | V | f_b f_a - f_a f_B \rangle
  \end{pmatrix}^{-1}
  \tag{4a}
  \]

- **Particle-hole residual interactions in self-consistent RPA**
  - derived from the second derivative of the energy functional
  - with rearrangement terms, if the meson-nucleon couplings are density-dependent
RHF+RPA in charge-exchange channel

- Particle-hole residual interactions
  - $\sigma$-meson: \[ V_\sigma(1, 2) = -[g_\sigma \gamma_0]_1 [g_\sigma \gamma_0]_2 D_\sigma(1, 2) \]  
  - $\omega$-meson: \[ V_\omega(1, 2) = [g_\omega \gamma_0 \gamma^\mu]_1 [g_\omega \gamma_0 \gamma_\mu]_2 D_\omega(1, 2) \]  
  - $\rho$-meson: \[ V_\rho(1, 2) = [g_\rho \gamma_0 \gamma^\mu \vec{\tau}]_1 \cdot [g_\rho \gamma_0 \gamma_\mu \vec{\tau}]_2 D_\rho(1, 2) \]
  - Pseudovector $\pi$-$N$ coupling: \[ V_\pi(1, 2) = -\left[ \frac{f_\pi}{m_\pi} \vec{\tau} \gamma_0 \gamma_5 \gamma^k \partial_k \right]_1 \cdot \left[ \frac{f_\pi}{m_\pi} \vec{\tau} \gamma_0 \gamma_5 \gamma^l \partial_l \right]_2 D_\pi(1, 2) \]
  - Zero-range counter-term of $\pi$-meson: \[ V_{\pi\delta}(1, 2) = g' \left[ \frac{f_\pi}{m_\pi} \vec{\tau} \gamma_0 \gamma_5 \gamma \right]_1 \cdot \left[ \frac{f_\pi}{m_\pi} \vec{\tau} \gamma_0 \gamma_5 \gamma \right]_2 \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad g' = 1/3 \]

- $\pi$-meson is included naturally.
- $g' = 1/3$ in the zero-range counter-term of $\pi$-meson is maintained for the sake of self-consistency.
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RHF+RPA for Gamow-Teller resonances

Gamow-Teller resonances in $^{48}$Ca, $^{90}$Zr, and $^{208}$Pb

GTR excitation energies can be reproduced in a fully self-consistent way.
GTR excitation energies and strength

GTR excitation energies in MeV and strength in percentage of the $3(N - Z)$ sum rule within the RHF+RPA framework. Experimental and the RH+RPA results are given for comparison. 

<table>
<thead>
<tr>
<th></th>
<th>$^{48}$Ca</th>
<th></th>
<th>$^{90}$Zr</th>
<th></th>
<th>$^{208}$Pb</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>energy</td>
<td>strength</td>
<td>energy</td>
<td>strength</td>
<td>energy</td>
<td>strength</td>
</tr>
<tr>
<td>experiment</td>
<td>~ 10.5</td>
<td>15.6 ± 0.3</td>
<td>19.2 ± 0.2</td>
<td>60-70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RHF+RPA</td>
<td>PKO1</td>
<td>10.72</td>
<td>69.4</td>
<td>15.80</td>
<td>68.1</td>
<td>18.15</td>
</tr>
<tr>
<td></td>
<td>PKO2</td>
<td>10.83</td>
<td>66.7</td>
<td>15.99</td>
<td>66.3</td>
<td>18.20</td>
</tr>
<tr>
<td></td>
<td>PKO3</td>
<td>10.42</td>
<td>70.7</td>
<td>15.71</td>
<td>68.9</td>
<td>18.14</td>
</tr>
<tr>
<td>RH+RPA</td>
<td>DD-ME1</td>
<td>10.28</td>
<td>72.5</td>
<td>15.81</td>
<td>71.0</td>
<td>19.19</td>
</tr>
</tbody>
</table>

The pion is not included in PKO2.
Physical mechanisms of GTR

- RH+RPA
  - no contribution from isoscalar mesons ($\sigma$, $\omega$), because exchange terms are missing.
  - $\pi$-meson is dominant in this resonance.
  - $g'$ has to be refitted to reproduce the experimental data.

- RHF+RPA
  - isoscalar mesons ($\sigma$, $\omega$) play an essential role via the exchange terms.
  - $\pi$-meson plays a minor role.
  - $g' = 1/3$ is kept for self-consistency.

Spin-dipole resonances

- Main peak can be reproduced by RHF+RPA \[\text{exp. Yako:2006}\]
- Energy hierarchy
  - RHF+RPA: \[E(2^-) < E(1^-) < E(0^-)\] agree with SHF+RPA \[\text{Fracasso:2007}\]
  - RH+RPA: \[E(2^-) < E(0^-) < E(1^-)\]

- Separating experimentally the different components from the total transition strength would be helpful to evaluate the theoretical predictive power, e.g.,
  - SDR in \(^{208}\)Pb \[\text{Wakasa:2010 arXiv}\] and \(^{16}\)O \[\text{Wakasa:2011}\]
A most recent $^{16}\text{O}(\vec{p},\vec{n})^{16}\text{F}$ experiment Wakasa et al., *PRC* 84, 014614 (2011)
Spin-dipole excitations by RHF+RPA

\[ ^{16}\text{O}(p,n)^{16}\text{F} \]

PKO1 (RHF+RPA)

- SDR in $T_-$ channel by RHF+RPA, where for $E_x$ the $0_{1}^{-}$ state is taken as reference. exp. Wasaka:2011

- In general, the $0^{-}$, $1^{-}$, and $2^{-}$ excited states are well reproduced within 1 MeV.
- The $0_{1}^{-}$, $1_{1}^{-}$, and $2_{1}^{-}$ triplets are found at $E_x \simeq 0$ MeV.
- The shoulder at $E_x = 5.86$ MeV and giant resonance at $E_x \simeq 7.5$ MeV are nicely reproduced. In particular, the shoulder state cannot be described by shell model calculations.
Spin-dipole excitations by RHF+RPA

\[ ^{16}\text{O(p,n)}^{16}\text{F} \]

PKO1 (RHF+RPA)

\[ 0^-, 1^-, 2^- \]

\[ 2^- \]

\[ 1^-, 2^-, 1^-, 2^- \]

\[ \text{total} \]

- SDR in \( T^- \) channel by RHF+RPA, where for \( E_x \) the \( 0^-_1 \) state is taken as reference. exp. Wasaka:2011

- The mixtures of \( 1^- \) and \( 2^- \) states at \( E_x \sim 9.5 \text{ MeV} \) and \( E_x \sim 12 \text{ MeV} \) are reproduced, and the former one is dominant by \( 2^- \) component, whereas the latter one is dominant by \( 1^- \) component.

- The \( 0^- \) resonances are predicted to be fragmented at \( 11 \sim 17 \text{ MeV} \) with the peak at \( E_x \sim 13.5 \text{ MeV} \).
2\(^{-}\) states at \(E_x \simeq 5.5\) MeV

- Particle-hole amplitudes \(X_{ph}^2\) for the 2\(^{-}\) states at \(E_x \simeq 5.5\) MeV.

<table>
<thead>
<tr>
<th>p-h configuration</th>
<th>(E_x = 5.23) MeV</th>
<th>(E_x = 5.54) MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\nu 1p_{1/2} \rightarrow \pi 4d_{5/2})</td>
<td>0.42</td>
<td>0.57</td>
</tr>
<tr>
<td>(\nu 1p_{3/2} \rightarrow \pi 1d_{5/2})</td>
<td>0.37</td>
<td>0.20</td>
</tr>
<tr>
<td>(\nu 1p_{1/2} \rightarrow \pi 4d_{3/2})</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>(\nu 1p_{3/2} \rightarrow \pi 2s_{1/2})</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>(\nu 1p_{1/2} \rightarrow \pi 3d_{3/2})</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>(\nu 1p_{1/2} \rightarrow \pi 1d_{5/2})</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\(\varepsilon(\nu 1p_{3/2}) = -20.98\) MeV, \(\varepsilon(\nu 1p_{1/2}) = -14.56\) MeV \\
\(\varepsilon(\pi 1d_{5/2}) = -1.89\) MeV, \(\varepsilon(\pi 4d_{5/2}) = 6.23\) MeV

- They are collective excitations.
- The most dominant particle-hole configuration \(\nu 1p_{1/2} \rightarrow \pi 4d_{5/2}\) is not considered in shell model calculations [Wakasa:2011].
SD excitations by RHF+RPA and RH+RPA

**RH+RPA results**

- The general pattern of $2^-$ excitations are similar to that of RHF+RPA calculations, except the peak at $E_x \approx 9.5$ MeV is missing.
- The $1^-$ resonances are predicted at $12 \sim 15$ MeV, somehow too high in energy by comparing to data.
- The $0^-$ resonances are predicted to be centralized at $10 \sim 12$ MeV, but not seen in experiments yet.

**Conclusion**

- By comparing with the experimental date, it is found that the self-consistent RHF+RPA calculations are more favored.
Differential inclusive cross sections of the reactions $^{16}\text{O}(\nu_e, e^-)^{16}\text{F}$ with $E_\nu = 35$ MeV as a function of the angle of the outgoing electron.

- Dominant contributions come from $J^\pi = 1^-, 2^-$ components.
- The electron emission is predominantly in the backward direction.
- $0^-$ component contributes to the forward angles.
- $0^-$ resonances in higher energy region lead to smaller neutrino-nucleus cross sections.
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- RHF+RPA

Spin-isospin Resonances

Summary
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★ A fully self-consistent charge-exchange relativistic RPA approach based on the RHF approach is established.

- Spin-isospin resonances
  - GTR excitation energies can be reproduced in a fully self-consistent way.
  - Isoscalar mesons ($\sigma$, $\omega$) are found to play an essential role via the exchange terms in GTR and SDR, while $\pi$-meson plays a minor role.
  - Fine structure of SD excitations in $^{16}$O can be well reproduced.
  - $J^\pi = 0^-$ component substantially contributes to the cross-section of $^{16}$O($\nu_e$, $e^-$)$^{16}$F at the forward angles.

Thank you!