Probing the high-density nuclear equation of state from heavy-ion collisions

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Outline

- Recent progress on studies of the nuclear equation of state (isospin symmetric and asymmetric)
- Transport model: Lanzhou Quantum Molecular Dynamics (LQMD) approach
  - Mean-field potential (isospin and momentum dependent)
  - Particle production ($\pi$, $\eta$, K, hyperons)
- Isospin emission at high baryon density
- Summary
Isospin asymmetric nuclear equation of state

\[ E(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^2) \]

\[ \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) \]

\( E_{\text{sym}} \) at supra-saturation densities

Supernova explosion, neutron star
Constraints of EoS from heavy-ion collisions

1. Isospin symmetric EoS: around saturation densities

Incompressibility: $K_0 = 9\rho_0^2 \left( \frac{d^2E}{d\rho^2} \right)_{\rho_0}$

- $K_0 = 231 \pm 5$ MeV
- PRL 82, 691 (1999)

Recent results:
- $K_0 = 240 \pm 10$ MeV
  - G. Colo et al.
  - U. Garg et al.

**Giant Monopole Resonance**

Frequency $f_{\text{GMR}} \propto \sqrt{K_0}$

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Incompressibility of Nuclear Matter from the Giant Monopole Resonance

D. H. Youngblood, H. L. Clark, and Y.-W. Liu

Cyclotron Institute, Texas A&M University, College Station, Texas 77843

(Received 30 July 1998)

E0 strength distributions in $^{92}$Zr, $^{116}$Sn, $^{164}$Sn, and $^{208}$Pb have been measured with multiple scattering of 240-MeV $\alpha$ particles between $0^\circ \leq \theta_{\text{lab}} \leq 6^\circ$ to greater precision than previously available. In Sn, Sn, and Pb, E0 strength was concentrated in approximately symmetric peaks, whereas in $^{92}$Zr it had a significant high-energy tail. Comparing with macroscopic calculations using the Gogny interaction, these and our previously reported results for $^{40}$Ca are consistent with a nuclear matter incompressibility of $231 \pm 5$ MeV. Previous data gave an average of 215 MeV and the value for different nuclei disagreed by up to 40 MeV. [50331-5037(98)08291-X]
2. Isospin symmetric EoS: at supra-saturation densities ($1\rho_0 < \rho < 3\rho_0$ from $K^+$ production in HIC’s around threshold energies) (J. Aichelin and C.M. Ko, PRL55, (1985) 2661)

G.Q. Li, C. M. Ko, PLB349(1995)405
BUU

C. Fuchs et al., PRL86(2001)1974
QMD
3. Isospin symmetric EoS: **at high densities** \((2\rho_0 < \rho < 5\rho_0\) using flow data from BEVALAC, SIS/GSI and AGS)


- Use constrained mean fields to predict the EOS for symmetric matter
  - Width of pressure domain reflects uncertainties in comparison and of assumed momentum dependence.

\[
P(\rho) = \rho^2 \left( \frac{\partial E}{\partial \rho} \right)_s
\]

The highest pressure recorded under laboratory controlled conditions in nucleus-nucleus collisions

High density nuclear matter
2 to 5\(\rho_0\)
4. Isospin asymmetric EoS: \textit{at sub-saturation densities} (0.1\(\rho_0<\rho<1.2\rho_0\))

\textit{Isospin observables:}

- Proton-nucleus elastic scattering in inverse kinematics
- Parity violating electron scattering studies of the n-skin in \(^{208}\text{Pb}\)
- n/p ratio of FAST, pre-equilibrium nucleons
- Double n/p ratios of isotopic reaction systems
- Isospin fractionation and isoscaling in nuclear multifragmentation
- Isospin diffusion/transport
- Neutron-proton differential flow
- Neutron-proton correlation functions at low relative momenta
- \(t/^{3}\text{He}\) ratio
- Hard photon production

Based on several complementary approaches with available data

\(E_{\text{sym}}(\rho)\approx31.6(\rho/\rho_0)^\gamma\ \text{MeV}, \ \gamma=0.69-1.05\)

\textbf{Refs:} B. A. Li et al., Phys. Rep. 464 (2008) 113; Chen/Ko/Li, PRL94, (2005) 032701; Li/Chen, PRC72, 064611(2005); Shetty et al., PRC75(07); PRC76(07); Tsang et al., PRL 102, 122701 (2009)
5. Isospin asymmetric EoS: at very low densities ($\rho < 0.1\rho_0$)

S. Kowalski, et al., PRC 75 (2007) 014601

6. Isospin asymmetric EoS: at supra-saturation densities ($\rho > 1.2 \rho_0$)

**Isospin observables:**

- $\pi^-/\pi^+$ ratio, $K^0/K^+$ ratio
- Neutron-proton differential transverse flow
- n/p ratio at mid-rapidity
- Double n/p, $\pi^-/\pi^+$, $K^0/K^+$ ratios of isotopic systems
- Nucleon elliptical flow at high transverse momenta
- n/p ratio of squeeze-out emission

... The information of high-density symmetry energy is poorly known, which is of importance in understanding the structure of neutron star, supernova explosion etc.
“Quantum” Molecular Dynamics (QMD)


Extension of the QMD model (isospin, low and high energies etc)

Isospin QMD (IQMD) by Nantes group (Hartnack, Aichelin);

And also by Chinese group, such as CIAE (Zhuxia Li et al., Beijing) and IMP (Lanzhou);

QMD (Fuchs et al., Tuebingen Uni, Germany);

Improved QMD (ImQMD) by Zhuxia Li group (CIAE, Beijing)

Ultra-relativistic QMD (UrQMD) by Frankfurt group

Lanzhou QMD (LQMD) for heavy-ion collisions from Coulomb barrier to 1A GeV energies
Lanzhou Quantum Molecular Dynamics (LQMD) model: physical purposes

- **Dynamics of low-energy heavy-ion collisions**
  (dynamical interaction potential, barrier distribution, neck dynamics, fusion/capture excitation functions etc)

- **Isospin physics at intermediate energies**
  (constraining nuclear symmetry energy at sub or supra saturation densities in HICs)

- **Medium effects in heavy-ion collisions and proton induced reactions**
  (probing the in-medium NN elastic and inelastic cross sections, properties of resonances $\Delta(1232)$, $N^*(1440)$, $N^*(1535)$) and mesons such as $\pi$, kaons, $\eta$)

- **Dilepton production ($e^+e^-$) at near threshold energies in heavy-ion collisions and proton induced reactions.**
The Hamiltonian in LQMD

\[ H = T + U_{\text{int}} + U_{\text{sh}}. \]

\[ U_{\text{int}} = U_{\text{coul}} + U_{\text{loc}} + U_{\text{sh}} + U_{\text{mom}} \]

\[ U_{\text{loc}} = \int V_{\text{loc}}(\mathbf{r}) d\mathbf{r} \]

\[ V_{\text{loc}}(\rho) = \frac{\alpha}{2} \frac{\rho^2}{\rho_0} + \frac{\beta}{\gamma + 1} \frac{\rho^{\gamma+1}}{\rho_0^\gamma} + \frac{g_{\text{sur}}}{2\rho_0} (\nabla \rho)^2 + \frac{g_{\text{sur}}^\text{iso}}{2\rho_0} [\nabla(\rho_n - \rho_p)]^2 + \]

\[ g_{\tau} \frac{\rho^{8/3}}{\rho_0^{5/3}} + E_{\text{sym}}^{\text{pot}}(\rho) \rho \delta^2 \]
Symmetry energy per nucleon in the LQMD model is derived from the Skyrme energy-density functional

\[ E_{\text{sym}}(\rho) = \frac{1}{3} \frac{\hbar^2}{2m} \left( \frac{3}{2} \pi^2 \rho \right)^{2/3} + a_{\text{sym}} \frac{\rho}{\rho_0} + b_{\text{sym}} \left( \frac{\rho}{\rho_0} \right)^\gamma + c_{\text{sym}} \left( \frac{\rho}{\rho_0} \right)^{5/3} \]

\[ a_{\text{sym}} = -\frac{1}{8} (2x_0 + 1) t_0 \rho_0, \quad b_{\text{sym}} = -\frac{1}{48} (2x_3 + 1) t_3 \rho_0^\gamma, \]

\[ c_{\text{sym}} = -\frac{1}{24} \left( \frac{3}{2} \pi^2 \right)^{2/3} \rho_0^{5/3} \left[ 3t_1 x_1 - t_2 (5x_2 + 4) \right]. \]

More clearly compared with other transport models, e.g., IBUU04, RBUU, SMF etc

\[ E_{\text{sym}}(\rho) = \frac{1}{3} \frac{\hbar^2}{2m} \left( \frac{3}{2} \pi^2 \rho \right)^{2/3} + \frac{1}{2} C_{\text{pot}} \left( \frac{\rho}{\rho_0} \right)^\gamma. \]

\[ E_{\text{sym}}(\rho) = \frac{1}{3} \frac{\hbar^2}{2m} \left( \frac{3}{2} \pi^2 \rho \right)^{2/3} + a \frac{\rho}{\rho_0} + b \left( \frac{\rho}{\rho_0} \right)^2 \]

with \( C_{\text{pot}}=38 \) MeV, \( a=37.7 \) MeV, \( b=-18.7 \) MeV which gives the \( E_{\text{sym}}=31.5 \) MeV at \( \rho=\rho_0=0.165 \).
Momentum dependent interaction potential in LQMD

\[
U_{\text{mom}} = \frac{1}{2\rho_0} \sum_{i,j,i\neq j} \delta_{\tau,\tau_i} \delta_{\tau_i,\tau_j} \int \int \int d\mathbf{p} d\mathbf{p}' d\mathbf{r} \, f_i(\mathbf{r}, \mathbf{p}, t) \ln(\epsilon(\mathbf{p} - \mathbf{p}')^2 + 1) f_j(\mathbf{r}, \mathbf{p}', t)
\]

with \[C_{\tau,\tau'} = C_{\text{mom}}(1 + x), \ C_{\tau,\tau'} = C_{\text{mom}}(1 - x) \ (\tau \neq \tau')\]

\[C_{\text{mom}} = 1.76 \text{ MeV}, \ \epsilon = 500 \text{ c}^2/\text{GeV}^2, \ x = -0.65. \] The parameters result in effective mass \[m^*/m = 0.75\] and \[m_n^* > m_p^*\] at \[\rho = \rho_0.\]

The symmetry energy in LQMD

\[
E_{\text{sym}}(\rho) = \frac{1}{3} \frac{\hbar^2}{2m} \left( \frac{3}{2} \pi^2 \rho \right)^{2/3} + E_{\text{sym}}^{\text{loc}}(\rho) + E_{\text{sym}}^{\text{mom}}(\rho)
\]
Nucleon effective mass normalized by vacuum mass as functions of density and isospin asymmetry
Density dependence of the potential part of nuclear symmetry energy at different mass splitting for hard (left panel) and supersoft (right panel) symmetry energies.
Different stiffness of the nuclear symmetry energy used in IBUU04 (by Bao-An Li, Refs Phys. Rev. C 69, 011603(R) (2004); Nucl. Phys. A 735, 563 (2004)) and LQMD (both models have included the isovector part of the momentum dependent interaction)
Momentum dependence of symmetry potential in nuclear reactions
Particle production in LQMD:

\( \pi \) and resonances (\( \Delta(1232) \), \( N^*(1440) \), \( N^*(1535) \))

\[ NN \leftrightarrow N \Delta, \quad NN \leftrightarrow NN^*, \quad NN \leftrightarrow \Delta \Delta, \]
\[ \Delta \leftrightarrow N \pi, \quad N^* \leftrightarrow N \pi, \quad NN \rightarrow NN\pi (s - \text{state}). \]

**Strangeness channels**

\[ BB \rightarrow BYK, \quad BB \rightarrow BBK\overline{K}, \quad B\pi \rightarrow YK, \]
\[ B\pi \rightarrow NK\overline{K}, \quad Y\pi \rightarrow N\overline{K}, \quad N\overline{K} \rightarrow Y\pi, \quad YN \rightarrow \overline{K}NN \]

And \( B(N, \Delta, N^*) \), \( Y(\Delta, \Sigma) \), \( K(K^0, K^+) \) and \( \overline{K}(K^0, K^-) \).

The isospin observables \( \pi^-/\pi^+, K^0/K^+, \Sigma^-/\Sigma^+ \) can be probes of high-density behavior of isospin asymmetric EoS
NN elastic and inelastic cross sections

Time evolutions of pion, kaon and sigma in the reaction $^{197}\text{Au} + ^{197}\text{Au}$ at 1.5A GeV by LQMD

Excitation functions of strangeness production and compared with K+ production of KaoS data (Phys. Rev. C 75, 024906 (2007))
In-medium modification of pion propagation: pion potential
Influence of pion optical potential on the $\pi^-/\pi^+$ yields in the $^{197}$Au+$^{197}$Au reaction

J. Xu, C. M. Ko and Y. Oh,
PRC 81 (2010) 024910

FIG. 4. (Color online) The $\pi^-/\pi^+$ ratio in Au + Au collisions at the beam energy of 0.4 A GeV for different values of nuclear symmetry energy parameter ($x = 0, 0.5, \text{ and } 1$) and the Migdal parameter $g' = 1/3, 0.4, 0.5, \text{ and } 0.6$. Results for $g' = \infty$ correspond to the case without the pion in-medium effects.
Isospin asymmetric EoS from heavy-ion collisions

\[ E(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^2) \]
\[ \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) \]

\[ E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + \frac{L}{3} \left( \frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2, \quad (\rho \neq \rho_0) \]

\[ E_{\text{sym}}(\rho_0) \approx 30 \text{ MeV} \] (LD mass formula: Myers & Swiatecki, NPA81; Pomorski & Dudek, PRC67)

\[ L \equiv 3\rho_0 \left. \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} \right|_{\rho=\rho_0} \] (Many-Body Theory: \( L : -50 \square 200 \text{ MeV; Exp: ???} \))

\[ K_{\text{sym}} \equiv 9\rho_0^2 \left. \frac{\partial^2 E_{\text{sym}}(\rho)}{\partial \rho^2} \right|_{\rho=\rho_0} \] (Many-Body Theory: \( K_{\text{sym}} : -700 \square 466 \text{ MeV; Exp: ???} \))

The isospin part of the isobaric incompressibility \( K_r \) of asymmetric nuclear matter
\[ K_r \approx K_{\text{asy}} = K_{\text{sym}} - 6L \text{(GMR : 320±180 MeV (Sharma et al., PRC38, 2562 (88)); } \\
\[ -566\pm1350 \square 34\pm159 \text{ MeV (Shlomo&Youngblood, PRC47,529(93)); } \\
\[ -550\pm100 \text{ MeV (T. Li et al, PRL99,162503(2007))} \)
Phenomenological calculations:

L.W. Chen, C.M. Ko, and B.A. Li
PRC72,064309 (2005); PRC76, 054316 (2007)

\[ \text{E}_{\text{sym}}(\rho) \text{ (MeV)} \]

\[ \rho \text{ (fm}^{-3}\text{)} \]

(a) NL RMF
(b) DD RMF
(c) PC RMF

Skyrme-Hartree-Fock
with 21 parameter sets

MDI interaction with \( x = 0 \)
MDI interaction with \( x = -1 \)
Microscopical calculations:

Z.H. Li et al., PRC74, 047304(2006)

Dieperink et al., PRC68, 064307(2003)

E_{lab}=0.4A GeV

π+/π^+

(N/Z)_{sys}

E_{lab}=1.5A GeV

π+/π^+

(N/Z)_{sys}

E_{sym} (MeV)

ρ/ρ_0

IBUU04: PRL102(2009)062502

E_{sym} (MeV)

ρ/ρ_0

E_{beam}(AGeV)

(N/Z)_{sys}

MDI interaction

APR

x = 1 (Gogny)

x = 0.5
Comparing both models for the kaon-nucleon interaction, Eqs. (8), (9) versus (4), (5), one sees that in the *OBE* model the nucleon–meson couplings enter explicitly in the kaon self energies, while in the *ChPT* approach the kaon mean field is determined by the parameters $\Sigma_{KN}$ and $C$. It is therefore helpful to study the isospin dependence of the *OBE* model for various parameterizations of the isovector EoS, i.e., the symmetry energy. As in previous works [10,14], we use the $NL$, $NL\rho$ and $NL\rho\delta$ prescriptions for the isovector mean-field. In the first one ($NL$) no isospin potential is included. $NL\rho$ contains only the $\rho$ meson, while $NL\rho\delta$ accounts for both, the $\rho$ and the $\delta$ mesons in the isovector potential. The symmetry energy becomes stiffer in the direction $NL \rightarrow NL\rho \rightarrow NL\rho\delta$, since one has to increase the $\rho$-meson coupling when the $\delta$ meson field is taken into account in the potential part of the symmetry energy.
Symmetry energy from elliptic flow in $^{197}$Au + $^{197}$Au

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FP1: weighted mean $\gamma = 1.01(21)$

FP2: weighted mean $\gamma = 0.98(35)$

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RBUU:
PRL97(2006)202301

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Dense UrQMD
Double n/p ratio at mid-rapidity ($|y/y_{\text{proj}}|<0.3$) as a probe of nuclear symmetry energy at 400 A MeV

$^{124}\text{Sn}+^{124}\text{Sn}$

$^{124}\text{Sn}+^{124}\text{Sn}/^{112}\text{Sn}+^{112}\text{Sn}$

(a) $b=1$ fm

(b) $b=6$ fm
Transverse momentum distribution of double $\pi^-/\pi^+$ ratio
Nucleon and cluster emission in the $^{197}$Au+$^{197}$Au reaction at 400 A MeV: flow at high baryon densities
High-density probes of symmetry energy from kaons

Exp: Lopez et al. FOPI, PRC75, 011901(R) (2007)

\[ \frac{K^0}{K^+} / \frac{K^0}{K^0} \text{ at } 1.528 \text{ AGeV} \]

RBUU: NPA832 (2010) 88

Fig. 5. $K^0/K^+$ ratio for different parameterizations of the isovector $EoS$ (NL, NL$\rho$, NL$\rho\delta$) and for the kaon potential (OBE, ChPT, IOBE, IChPT). The considered reaction is a central ($b = 0$ fm) Au + Au heavy-ion collision at 1 A GeV incident energy.
Summary:

A isospin and momentum dependent transport model (LQMD) for simulating heavy-ion collisions from near barrier to relativistic energies is introduced. The isospin physics, in particular high-density behavior of EoS, is investigated.

Further subjects of the LQMD model:

(a) Based on the model, the high-density behavior of the nuclear symmetry energy will be further constrained from strangeness production, such as $K^0/K^+$, $\Sigma^-/\Sigma^+$ etc.

(b) Influence of the in-medium properties of resonances on the isospin observables is to be performed.

(c) Probing the in-medium properties from the dilepton spectra ($e^+e^-$) in proton induced reactions and in heavy-ion collisions.