

The implications of a holographic universe for quantum information science and the nature of physical law

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Abstract

Several lines of evidence suggest that the total information content of the observable universe is bounded by a finite number given by the area of a cosmological surface divided by the Planck area. This is referred to as the holographic principle. The current bound is roughly 10^{122} bits, but in the past it was smaller, varying like t^2 in the early universe. Although the bound is too large today to affect most of everyday physics, it does have profound implications for highly complex systems and for cosmology. For example, the project to build a useful quantum computer, which is projected to exploit states possessing exponential complexity, comes into conflict with the information bound at a level of entanglement of about 400 qubits, suggesting a breakdown of unitary evolution at this threshold, possibly associated with the emergence of classicality. If the information bound is applied to the quantum vacuum, it yields an energy density close to the observed density of cosmological dark energy. However, because the bound is time-dependent, the vacuum energy will vary in time too, and consistency with energy conservation then demands that G or c varies with time over cosmological time scales. Further sweeping implications follow if one adopts the philosophy that information is primary ('the universe is a computer'), and that the laws of physics do not exist in a transcendent Platonic realm of perfect mathematical forms and operations, as is conventionally supposed, but are fundamentally tied to the real physical universe, with its finite age and resources, and subject to the holographic information bound. I suggest a generalization of the information bound based on this point of view, formulated in terms of algorithmic information theory, and briefly mention some of consequences for black holes and the inflationary universe scenario.

Why should I believe in a real number if I can't calculate it, if I can't prove what its bits are, and if I can't even refer to it? ...The real line from 0 to 1 looks more and more like a Swiss cheese.

Gregory Chaitin (2005)

Whereof one cannot speak, thereof one must remain silent.

Ludwig Wittgenstein (1921)

1. Black hole entropy and the Bekenstein bound

A new and surprising link between physics and information followed a landmark development in theoretical physics in 1970, when Jacob Bekenstein proposed that the surface area of a black hole serves as a measure of its entropy (Bekenstein 1973). This unexpected relationship was suggested on the basis of quantum mechanics applied to the black hole, and it was confirmed a few years later by Stephen Hawking in a detailed calculation (Hawking 1975). For an uncharged, non-rotating black hole, the Bekenstein-Hawking relationship is

$$S = 4\pi kGM^2/\hbar c^3 = \frac{1}{4}A, \quad (1)$$

where S , M and A are the entropy, mass and area of the black hole respectively, and the other symbols have their usual meanings as various fundamental constants of nature.

The fact that the entropy is a function of black hole *area*, as opposed to volume, is deeply significant. In the case of a laboratory gas, for example, its entropy will be simply additive: twice the volume of a (homogeneous) gas will have twice the entropy. Evidently, when gravitation enters the picture, the rules of the game change fundamentally. Entropy has long been regarded as a measure of information I (or information loss), through the relationship

$$S = k\log_2 I \quad (2)$$

so the Bekenstein-Hawking formula relates the total information content of a region of space to the area of the surface encompassing that volume. The information inside a black hole is lost because an observer in the external region cannot access it on account of the fact that the surface of the hole is bounded by an event horizon. (There remains an unresolved issue about whether the information is permanently lost, or just rendered inaccessible until the black hole eventually evaporates. I shall not consider that issue further in this paper.) A useful way to think about Eq. (1) is to define the Planck length $L_P \equiv (G/\hbar c^3)^{1/2}$ as a fundamental unit, and note that, using Eq. (2), the information of the black hole is simply one quarter of the horizon area in Planck units.

Early on, Bekenstein sought to generalize his result by postulating that Eq. (1) serves as a *universal* bound on entropy (or information content) applicable to *any* physical system (Bekenstein, 1977). That is, the information content of a physical system can never, it is claimed, exceed one quarter of the area of its encompassing surface. The black hole saturates the Bekenstein bound, and represents the maximum amount of information that can be packed into the volume occupied by the hole, as befits the equilibrium end state of a gravitating system. A simple argument in support of the universal Bekenstein bound is that if a system confined to a certain region of space possessed an information content in excess of the bound, one could then add some matter and induce this system to undergo gravitational collapse to a black hole, thereby reducing its entropy and violating the second law of thermodynamics (suitably generalized to include event horizon area).

The idea of associating entropy and information with horizon area was soon extended to include *all* event horizons, not just those surrounding black holes. For example, if the universe becomes dominated by dark energy, which is what current astronomical observations suggest, it will continue to expand at an accelerating rate (dark energy acts as a sort of antigravity force). This creates a *cosmological* event horizon, which may be envisaged as a roughly spherical surface that bounds the region of the universe to which we have causal and informational access. A similar horizon characterizes the period of inflation, widely believed to have occurred at about 10^{-34} s after the big bang. Generalizations of horizon entropy have been proposed for cosmological horizon area too, with de Sitter space (a universe subject to dark energy alone) saturating the Bekenstein bound (Gibbons and Hawking 1977, Bousso 1999, Davies and Davis 2003). A number of calculations support this proposal.

Based on the foregoing ideas, 't Hooft (1993) and Susskind (1995) have proposed the so-called *holographic principle*, according to which the information content of the entire universe is captured by an enveloping surface that surrounds it. The principle states that the total information content of a region of space cannot exceed one quarter of the surface area that confines it, and that this limit is attained in the case of the cosmological event horizon. A simple calculation of the size of our universe's event horizon today based on the size of the event horizon created by the measured value of dark energy gives an information bound of 10^{122} bits:

$$I_{\text{universe}} \leq 10^{122}. \tag{3}$$

According to the holographic principle, this huge number represents an upper bound on the information content of the universe.

A similar number has been derived by Lloyd (2002), using a slightly different argument. Lloyd asks: what is the total number of bits the universe can have processed since its origin in a big bang? This number will be finite because of the finite age of the universe, the finite rate of information processing due to basic quantum mechanical and thermodynamics limitations, and the finite speed of light. The latter creates a particle horizon (in effect, a surface that defines the volume of space to which we have causal

access at this time). The fact that Lloyd derives a similar limit, 10^{122} bits, as implied by the holographic principle, stems from the well-known coincidence that the density of dark energy is roughly the same at the current cosmological epoch as the density of other forms of energy.

If the information bound on the universe is taken seriously, it has sweeping implications for the nature of the universe and for the nature of physical law. Taken at face value, the holographic principle implies that the universe is in a sense two-dimensional. Evidently the physical processes taking place within the volume of the universe can be captured by a set of rules applied to a cosmological surface. The information bound on this surface implies that the number of effective degrees of freedom in the universe is finite, albeit enormous, and that the Hilbert space describing its quantum dynamics is finite-dimensional (see, for example, Thomas 2002). Let me illustrate how this restriction affects physics and cosmology with some examples.

2. A cosmological information bound could solve the dark energy problem

A straightforward explanation for dark energy is that it is simply the energy of the quantum vacuum (see, for example, Birrell and Davies 1982). For a massless scalar field confined to a cube of space of linear dimension L , the energy density ρ of the vacuum is given by standard quantum field theory:

$$\rho = \frac{1}{2} \hbar c L^{-1} \sum_{\mathbf{k}} \omega, \quad (4)$$

where the sum is taken over all the field modes of momentum \mathbf{k} . The right hand side of Eq. (2) diverges like $\sim \omega^4$ as $\omega \rightarrow \infty$. It may be rendered finite by imposing a cut-off in the summation. A natural cut-off is provided by the Planck frequency, which incorporates only the fundamental constants already present in the theory: \hbar , c and G . Using this cut-off, Eq. (4) yields a vacuum energy density of 10^{114} Jm^{-3} , which is some 10^{122} times the observed dark energy density. This staggering discrepancy between theory and observation has been known for many years, and is known as the dark energy (or cosmological constant) problem, and is one of the main outstanding challenges to physical theory.

The occurrence of the same factor 10^{122} in this discrepancy as in the cosmological information bound is a clear pointer to an alternative explanation for dark energy, and indeed, inequality (3) provides a second natural cut-off for the summation in Eq. (4). Rewriting (4) in terms of modes,

$$\rho \approx \hbar c L^{-4} \sum n^4. \quad (5)$$

I now argue that the sum $\sum n^4$ should be bounded by inequality (3). Taking L to be the horizon radius (roughly a Hubble radius) and $\sum n^4 \sim 10^{122}$, we may then evaluate the vacuum energy density to be

$$\rho \approx 10^9 \text{ Jm}^{-3} \approx \rho_{\text{observed}}. \quad (6)$$

The same result may be derived in a completely different way, by imposing the condition on the vacuum energy that at every scale of size L , the energy density must not exceed the level at which the total mass within a volume L^3 exceeds the mass of a black hole of size L , otherwise the vacuum energy would presumably undergo gravitational collapse. This requirement may be expressed as follows:

$$\rho c^2 L^3 < M_{\text{bh}}(L). \quad (7)$$

Substituting the right hand side of Eq. (4) for ρ we obtain, to an order or magnitude,

$$G\hbar\omega^4 L^3/c^7 < L \quad (8)$$

or

$$\rho < c^4/GL^2. \quad (9)$$

Taking L to be the Hubble radius, inequality (9) may be re-cast in the following suggestive form:

$$\rho < (\rho_{\text{P}}\rho_{\text{H}})^{1/2} \approx 10^9 \text{ Jm}^{-3} \approx \rho_{\text{observed}} \quad (10)$$

where ρ_{P} is the Planck energy density and ρ_{H} is the Hubble energy density, defined to be the energy density of a single quantum in a Hubble volume with a wavelength equal to the Hubble radius. This result has been noted before (see, for example, Padmanabham 2004).

This remarkable result – that the cosmological information bound explains the magnitude of the dark energy – comes at a price, however. The same reasoning may be applied to the *pressure* of the vacuum, p , which for a massless scalar field is

$$p = -\frac{1}{2}\hbar c L^{-1} \sum \omega, \quad (11)$$

i.e. $p = -\rho$, which is the necessary equation of state for the vacuum energy to play the role of dark energy. The information bound (3) is not a fixed number. Rather, it varies with time. The time-dependence of I_{universe} will depend on the precise formulation of the information bound. For example, Lloyd's calculation using the particle horizon bound yields (Lloyd 2002)

$$I_{\text{universe}} \sim t^2. \quad (12)$$

The holographic bound is based on an event horizon, the radius of which is defined as follows:

$$R_{\text{h}} = a(t) \int_t^{\infty} dt'/a(t') \quad (13)$$

where $a(t)$ is the cosmological scale factor. If the density of dark energy remains constant, R_h will asymptote to a constant value at late times. But at early times, $R_h \sim t^2$ for a Friedmann-Robertson-Walker model universe. Thus in both cases, Eq. (12) expresses the time dependence of the information bound in the early universe. Hence the cut-off in the summation in both Eqs. (4) and (11) will be time-dependent, so the dark energy will also be time-dependent. This raises an immediate difficulty with the law of energy conservation:

$$pda^3 + d(\rho a^3) = 0 \tag{14}$$

which can be satisfied for a time-dependent p and ρ only if there is some compensatory change, e.g. G and/or c vary with time. There is a substantial literature on such holographic cosmological models, including comparison with observations, which I shall not review here (see, for example, Guberina, Horvat and Nicolici 2006; Hsu 2004; Li 2004, Thomas 2002).

3. A generic quantum computer may violate the cosmological information bound

A further transformation in our understanding of information came with the recognition that because nature is fundamentally quantum mechanical, the rules for information processing at the quantum level differ not only in the technical details but in their very conceptual basis. In conventional (classical) information theory, the basic unit is the bit, or binary choice, usually symbolized by 0 and 1. In quantum mechanics, the bit is replaced by a more abstract entity: the qubit. When humans read out the information content of a quantum system, they appropriate only bits – the act of read-out collapses qubits into bits. But the importance of quantum information dynamics is that in an isolated unobserved quantum system, the qubits generally evolve in a manner completely different from the classical case, involving the whole panoply of quantum weirdness, including, most crucially, superposition and entanglement. It is this feature that has commended quantum information science to governments and business by holding out the promise of large-scale quantum computation. By exploiting qubit dynamics, a quantum computer would represent an unprecedented leap in computational power. A review of the field may be found in Nielsen and Chuang (2000).

The key to quantum computation lies with the exponential character of quantum states. Whereas a classical binary switch is either on (1) or off (0), a quantum system can be in a superposition of the two. Furthermore, a multi-component quantum system can incorporate entanglement of spatially separated subsystems. Combining these two properties implies that an n -component system (e.g. n atoms) can have 2^n states, or components of the wave function, that describe the system. If it were possible to control all the components, or branches, of the wave function simultaneously, then the quantum system would be able to process information exponentially more powerfully than a classical computer. This is the essence of the quantum computation industry. So far, about a dozen entangled qubits have been constructed, but the goal is to link many thousands, or even millions, and permit them to evolve quantum mechanically, as isolated as possible from the decohering effect of the environment, and using error

correction protocols where disturbances afflict the system. In the final step, the result of the quantum computation is read out.

Because the complexity of an entangled state rises exponentially with the number of qubits (which is its virtue), large-scale quantum information processing comes into conflict with the cosmological information bound implied by the holographic principle. Specifically, a quantum state with more components than about $n = \log_2 I_{\text{universe}}$ will require more bits of information to define it than can be accommodated in the entire observable universe! Using the bound given by inequality (3), this yields a limit of approximately $n = 400$. In other words, an entangled state of more than about 400 particles will have a quantum state with more components than I_{universe} evolving in a Hilbert space with more dimensions than I_{universe} . The question therefore arises of whether this violation of the information bound (3) signals a fundamental physical limit. It seems to me that it must.

On the face of it, the limit of 400 particles is stringent enough to challenge the quantum computation industry, in which a long-term objective is to entangle many thousands or even millions of particles and control the evolution of the quantum state to high precision. The foregoing analysis, however, contains a possible loophole. First, note that the dimensionality of the (non-redundant part of the) Hilbert space is not an invariant number: by changing the basis, the number might be reduced. So specifying the complexity of a quantum state using the dimensionality of the Hilbert space can be misleading. A more relevant criterion is the number of independent parameters needed to specify inequivalent n -component quantum systems. This problem has been addressed, but it is a difficult one on which only limited progress has so far been made (see, for example, Linden and Popescu 1998).

A more subtle issue concerns the specific objectives of the quantum computation industry, which is not to control the dynamical evolution of *arbitrary* entangled quantum states, but an infinitesimal subset associated with mathematical problems of interest, such as factoring. It is trivially true that it is impossible to prepare, even approximately, a state containing more than 10^{122} truly independent parameters because it is impossible to even specify such a state: there are not enough bits in the universe to contain the specification. Almost all states fall into this category of being impossible to specify, prepare and control. So in this elementary sense, generic quantum computation is obviously impossible. Less obvious, however, is whether the subset of states (of measure zero) of interest to the quantum computing industry is affected by the cosmological information bound, for even if it is the case that the number of independent amplitudes exceeds 10^{122} , there may exist a compact mathematical algorithm to generate those amplitudes. (The algorithm for generating the amplitudes that specify the initial state should not be confused with the algorithm to be executed by the quantum computer dynamics.) For example, the amplitudes of the quantum computer's initial state could be the (unending) digits of π , which can be generated by a short algorithm. That is, the set of amplitudes may contain an unbounded number of bits of information, but a finite (and even small) number of bits might be sufficient to define the generating algorithm of the amplitude set. So if the information bound on the universe is interpreted as an upper limit on the

algorithmic information (as opposed to the Shannon information), then a measure-zero subset of initial states can be specified without violating the cosmological information bound. But this loophole leaves many unanswered questions. For example, a mathematical specification is one thing, a physical process to implement that specification – and to do so in an acceptable period of time – is another. To take the cited example, it is far from clear that there exists *any* physical process that can create an entangled quantum state in which the amplitudes (enumerated in some sequence) are the digits of π . And even if this further problem is satisfactorily addressed, one has to confront the fact that as the initial state evolves, and the amplitudes change, so the set of amplitudes may not remain algorithmically compressible. To be sure, a unitary evolution of an initially algorithmically compressible state will, by definition, preserve algorithmic compressibility (because the unitary operation is an algorithm). But such a pure system is unstable: the inevitability of random errors due to the fact that the quantum system is not closed will raise the algorithmic complexity, and seemingly raise it above the bound (3) in pretty short order. This uncovers a deeper set of issues, which is whether a quantum state that cannot be specified, and is in principle unknowable, and the amplitude set of which exceeds the total information capacity of the universe, may nevertheless still be said to exist and conform to physical law. I will defer a discussion of this topic until the final section.

I have been asked what, exactly, would go wrong if one tried to build and operate a quantum computer with, say, 500 entangled qubits. First let me make a general point. In science, one always has to distinguish between mathematical possibility contained in a theory, and physical possibility. For example, general relativity contains mathematical models with closed timelike world lines, but these may be inconsistent with cosmological boundary conditions or some other global requirement (Davies 2001). So the fact a unitary transformation that implements a desirable quantum computation may exist mathematically does not necessarily mean it can be implemented physically, even in principle. And in fact, a *prima facie* example would seem to be the expectation that the resources needed to prepare an initial quantum state are expected to grow with its complexity, and would require more and more of the surrounding universe to be commandeered, and more yet for the error correction of its evolution. Inevitably, the gravitational effects of the commandeered matter will eventually become important. Before the complexity of the state reached the cosmological bound of 10^{122} , the entire resources of the observable universe would necessarily be exhausted. Thus, almost all quantum initial states, and hence almost all unitary transformations, seem to be ruled out by the cosmological constraint (3) (if one believes it). It is important to realize, however, that this restriction may not be an impediment to preparing an algorithmically simple state, providing a physical mechanism can be found to implement the preparation algorithm. These criteria will undoubtedly be satisfied for the (very limited) examples of known quantum algorithms, such as Shor's algorithm for factorization, which is algorithmically simple by definition, since its input state can be specified and there is a simple association between the input data and the initial quantum state. What is less clear is whether this ease of preparation of the initial state is representative of a broad class of problems of interest, or confined to a handful of special cases.

A more radical conjecture of what might ‘go wrong’ concerns the subsequent evolution of the state, which entails an escalation of the algorithmic complexity through the cosmological information bound due to random errors in the manner I mentioned above. Under these circumstances, it may be that the unitary evolution of the state actually breaks down (over and above the breakdown caused by tracing out the degrees of freedom associated with the errors caused by environmental disturbances). This would manifest itself in the form of an additional source of errors, ultimately of cosmological origin, in a manner such that all error-correcting protocols applied to these errors would fail to converge. What I am suggesting here seems to be close to the concept of unavoidable intrinsic decoherence proposed by Milburn (1991, 2006). Some clarification of these issues may emerge from the further study of the recent discovery that the entropy of quantum entanglement of a harmonic lattice also scales like area rather than volume (Cramer and Eisert 2006), which would seem to offer support for the application of the holographic principle to entangled states. It would be good to know how general the entanglement-area relationship might be.

4. The emergence of classicality in sufficiently complex quantum systems

It is tempting to speculate that the departure from unitary evolution predicted by imposing the cosmological information bound is somehow associated with the emergence of classicality in quantum systems. A drawback of the so-called collapse models of the quantum-classical transition is the need to introduce additional ad hoc parameters and to adulterate quantum mechanics with small nonlinear terms. For example, in the GRW scheme, new fundamental units of frequency and size are proposed:

$$f = 10^{-16} \text{ s}^{-1}$$

$$d = 10^{-5} \text{ cm.}$$

The idea that classicality may emerge at a certain threshold of *complexity*, as opposed to mass or size, seems to have been largely overlooked (with the exception of a general conjecture by Leggett (1984)). One virtue of the complexity threshold proposal is that complexity is a dimensionless quantity, and a natural measure of complexity is given to us from cosmology, namely, the information bound (3). My proposal, therefore, is that classicality emerges in a system requiring more than I_{universe} bits of information to specify it. It is interesting to note that this restriction on the applicability of unitary quantum evolution will not affect the theory of quantum cosmology, in which the universe originates as some sort of quantum nucleation event, because that process may be described by a simple wave function.

5. The status of the laws of physics

The real reason to study quantum computing is not to learn other people's secrets, but to unravel the ultimate Secret of Secrets: is our universe a polynomial or an exponential place?

Scott Aaronson (2005)

Most theoretical physicists are by temperament Platonists. They consider that the laws of physics are perfect idealized mathematical forms and operations that really exist, but occupying an abstract realm transcending the physical universe. Thus the project of quantum cosmology, for example, is predicated on the assumption that the laws of quantum mechanics and (say) general relativity or string/M theory, exist independently of the universe, and may therefore be invoked to explain how the universe came to exist from nothing. In other words, the laws affect the universe, but the universe does not affect the laws. This fundamental asymmetry, however, seems eccentric when the universe is viewed in terms of information processing.

The traditional logical dependence of laws, states of matter and information is

A. laws of physics \rightarrow matter \rightarrow information.

Thus, conventionally, the laws of physics form the absolute and eternal bedrock of physical reality, and cannot be changed by anything that happens in the universe. Matter conforms to the 'given' laws, while information is a derivative, or secondary property having to do with certain special states of matter. But several physicists have suggested that the logical dependence should really be as follows:

B. laws of physics \rightarrow information \rightarrow matter.

In this scheme, often described informally by the dictum 'the universe is a computer,' information is placed at a more fundamental level than matter. Nature is treated as a vast information-processing system, and particles of matter are certain special states which, when interrogated by, say, a particle detector, extract or process the underlying quantum state information so as to yield particle-like results. It is an inversion famously encapsulated by Wheeler's pithy phrase 'It from bit' (Wheeler 1994). Treating the universe as a computer has been advocated by Fredkin (1990), Lloyd (2002, 2006) and Wolfram (2002) among others.

An even more radical transformation is to place *information* at the base of the logical sequence, thus

C. information \rightarrow laws of physics \rightarrow matter.

The attraction of scheme C is that, after all, the laws of physics are informational statements. In the orthodox scheme A, it remains an unexplained concordance that the

laws of physics are mathematical/informational, a mystery flagged by Wigner in his famous paper ‘The unreasonable effectiveness of mathematics in the physical sciences’ (Wigner 1960).

For most purposes the order of logical dependence does not matter much, but when it comes to the information bound on the universe, one is forced to confront the status of information: is it ontological or epistemological? If information is simply a description of *what we know* about the physical world, as is implied by Scheme A, there is no reason why Mother Nature should care about the limit (3). Or, to switch metaphors, the bedrock of physical reality according to Scheme A is sought in the perfect laws of physics, which live elsewhere, in the realm of the gods – the Platonic domain they are held by tradition to inhabit, where they can compute to arbitrary precision with the unlimited amounts of information at their disposal. The Platonic realm is the ‘real reality,’ according to orthodoxy, while the world of information is but the shadow on Plato’s cave. But if information really does underpin physical reality – if it, so to speak, occupies the ontological basement – (as is implied in Scheme C and perhaps B) then the bound on I_{universe} represents a fundamental limitation on *all reality*, not merely on states of the world that humans perceive.

To see how this plays out in real physics, consider a simple quantum superposition of two eigenstates φ_1 and φ_2 :

$$\psi = \alpha_1\varphi_1 + \alpha_2\varphi_2. \tag{15}$$

The amplitudes α_1 and α_2 are complex numbers which, in general, demand an *infinite* amount of information to specify them precisely (envisage them written as an infinite binary string). If one believes that this information is ‘merely epistemological,’ and that the mathematically idealized laws of physics are the true ontological reality (Scheme A), then infinitely information-rich complex numbers α_1 and α_2 exist contentedly in the Platonic realm of the gods, where they can be subjected to infinitely precise idealized mathematical operations such as unitary evolution. And the fact that we humans cannot, even in principle, and even by commandeering the entire observable universe, track those operations, is merely an epistemological handicap. So in Scheme A, there is no further implication of the information bound (3). In short, A says: The universe does not compute in the (resource-limited) universe; it computes in the (infinitely resourced) Platonic realm.

But if information is ontological, as for example in the heretical Scheme C, then we are obliged to assume that ‘the universe computes in the universe,’ and there isn’t an infinite source of free information in a Platonic realm at the disposal of Mother Nature. In that case, the bound on I_{universe} applies to *all* forms of information, including such numbers as α_1 and α_2 in Eq. (12), and to the dynamical evolution of the state vector ψ , and is not merely a bound on the number of degrees of freedom in the universe, or on the dimensionality of Hilbert space. Rolf Landauer was a strong advocate of the view that ‘the universe computes in the universe,’ because he believed that ‘information is physical’. He summed up his philosophy as follows (Landauer 1967):

‘The calculative process, just like the measurement process, is subject to some limitations. A sensible theory of physics must respect these limitations, and should not invoke calculative routines that in fact cannot be carried out.’

In other words, in a universe limited in resources and time – a universe subject to the information bound (3) in fact – concepts like real numbers, infinitely precise wave function coefficients, differentiable functions, or the unitary evolution of a wave function – are a fiction: a useful fiction to be sure, but a fiction nevertheless. Landauer’s proposal that our theories should be constrained by the resources of the universe has been developed in recent years by Benioff (2003).

Adopting Landauer’s position, if one were to demand that the total information needed to specify a quantum state is bounded by I_{universe} , there would be an *intrinsic* degree or error, or uncertainty, involved in specifying the amplitudes. In practice, of course, one cannot in any case prepare a quantum state with infinitely-precise amplitudes: there will always be some experimental error. Given the enormous value of I_{universe} the consequences of this intrinsic source of error will normally be swamped by the practical limitations involved in preparing almost all quantum states. But this conclusion cannot be drawn in the case of quantum computation involving a large number of qubits, because of the exponential nature of quantum entanglement and superposition.

A rigorous formulation of the effect of the cosmological information bound on the quantum dynamics of complex systems needs to take note of the following points. Some numbers (e.g. π) may be specified by the output of a compact algorithm that contains rather little information. Although the set of such algorithmically compressible numbers is of measure zero in the set of real numbers, it is nevertheless rich enough to accommodate all that humans need to describe physics in practice. Secondly, as I discussed in Section 3, a given quantum state may require a set of n amplitude parameters to define it, but there may exist a compact algorithm defined by a number of bits $\ll n$ which generates that set, or a subset. It seems reasonable that such states, which may be physically complex but algorithmically simple, should not be constrained by (3). These considerations suggest that the information bound (3) should be formulated in terms of algorithmic information rather than Shannon information. The algorithmic information measure of a binary string X is defined as

$$H(X) = -\ln P(X) + O(1) \tag{16}$$

where $P(X)$ is the probability that the proverbial monkey typing randomly on a typewriter will generate a program which, when run on a universal Turing machine, will output X . Applied to the amplitude set $\{\alpha_i\}$ of a generic quantum state (plus any ancillary information needed to specify the state, such as constraints), the cosmological information bound (3) may be expressed as follows:

$$H(\{\alpha_i\}) < A_{\text{holo}}/L_P^2 \tag{17}$$

where A_{holo} is the area of the appropriate holographic surface (e.g. a cosmological event horizon). Inequality (17) is a stronger constraint than (3), appropriate to the interpretation of information as ontological and fundamental, and therefore including not merely a head-count of the degrees of freedom, but the algorithmic information content of all the specifying parameters of the state too. This ‘extra’ burden on the bound will reduce somewhat the dimensionality of the Hilbert space at which ‘something funny’ will happen.

Some consequences

The stronger version of the information bound, inequality (17), will apply to all forms of information, not merely quantum information. For most physical systems it will not represent a severe constraint, but careful attention must be given to systems involving exponentiation, for then the bound is easily saturated in one of the system variables. In the colourful words of Aaronson (2005), ‘Schrödinger’s cat is out of the bag – and all of us are being forced to confront the exponential Beast that lurks inside our current picture of the world.’

I have already discussed the example of complex quantum systems. Other examples come readily to mind. Classical statistical mechanics involves Poincaré recurrence times of duration exponential in the number of participating particles. These recurrences are a fiction, according to the point of view being expounded here. Chaos theory involves exponential sensitivity to initial conditions: it is an unsolved problem of whether the theory is ‘overtaken’ by quantum effects before or after the bound (17) kicks in.

Another example is provided by general relativity and cosmology, where surfaces of infinite red shift such as horizons are a familiar feature. Also, the inflationary universe scenario is based on a period of exponential growth in the cosmological scale factor. An important consistency check on the holographic theory concerns the derivation of black hole radiance, the starting point for holographic reasoning, on which Bekenstein’s ideas depend. Hawking’s original derivation of the thermal nature of black holes (Hawking 1975) involved an argument in which outgoing modes of a quantum field are propagated back in time to the in region prior to the collapse phase, and a suitable Bogoliubov transformation is computed. The effect of the gravitational field of the collapsing body is to exponentiate the wavelength of the outgoing modes, and this appears as an exponential phase factor in the modes; conversely, modes in the out region are exponentially blue shifted. The Bogoliubov transformation involves an integral taken over all field modes up to infinite frequency, which then yields the thermal nature of the spectrum of particles created by the black hole. But according to the information bound, the integral must be cut off when the exponential phase factor saturates the bound (17). The result of this is to replace the steady thermal radiance of the black hole with a brief pulse of radiation (Jacobson 1993). This is a familiar problem in the theory: Hawking’s Bogoliubov transformation includes an integration over trans-Planckian modes, so if a cut-off is imposed at, say, the Planck frequency then the thermal radiance goes away. Fortunately, the Hawking effect may be derived in several different ways, and although a rigorous treatment of this problem has not been carried out, there is good circumstantial evidence

that the essentially thermal nature of black hole radiation will not be affected by truncating the high-frequency modes (Jacobson 1993). Similar considerations apply to the thermodynamic character of cosmological horizons.

Finally, let me turn to inflation. It is a key feature of the information bound that it is time-dependent. In the past, the bound was smaller, and its effects on physics would have been greater. During the very early universe, the effects could have been significant, and may have left a trace on the structure of the universe that could be used to test the existence of the bound. Inflation is a brief episode of exponential expansion thought to have occurred at about 10^{-34} s after the big bang. At that time, the horizon size was about 10^{-47} cm, yielding a horizon area of about 10^{-19} Planck areas. The information bound (17) then implies a bound on the inflation factor

$$a(t_{\text{after}})/a(t_{\text{before}}) < 10^{19}. \quad (18)$$

Guth's original proposal was for an inflation factor at least 10^{20} , so (given the rough-and-ready nature of the calculation) the information bound is consistent with inflation, but only marginally so, and a more detailed analysis may suggest observable consequences.

There is a more comprehensive consistency check that I shall not consider in this paper. The information bound was derived using quantum field theory, but that same bound applies to quantum field theory. Ideally one should derive the bound using a self-consistent treatment. If one adopts the philosophy suggested in the previous section – that information is primary and ontological – then such a self-consistency argument should be incorporated in a larger program directed at unifying mathematics and physics. If, following Landauer, one accepts mathematics is meaningful only if it is the product of real computational processes (rather than existing independently in a Platonic realm) then there is a self-consistent loop: the laws of physics determine what can be computed, which in turn determines the informational basis of those same laws of physics. Benioff (2002) has considered a scheme in which mathematics and the laws of physics co-emerge from a deeper principle of mutual self-consistency, thus addressing Wigner's question of why mathematics is so 'unreasonably effective' in describing the physical world. I have discussed these deeper matters elsewhere (Davies 2006).

Acknowledgments

I should like to thank Scott Aaronson, Ted Jacobson, Gerard Milburn, William Phillips, Sandu Popescu and Leonard Susskind for helpful comments, conversations and guidance.

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