

The Weak Gravity Conjecture

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based on paper by M. Li, W. Song and T. Wang

[hep-th/0601137](#)

and paper by Q. G. Huang, M. Li and W. Song

[hep-th/0603127](#)

Motivation

- One of the central problems in string theory is whether it can make predictions verifiable by experiments.

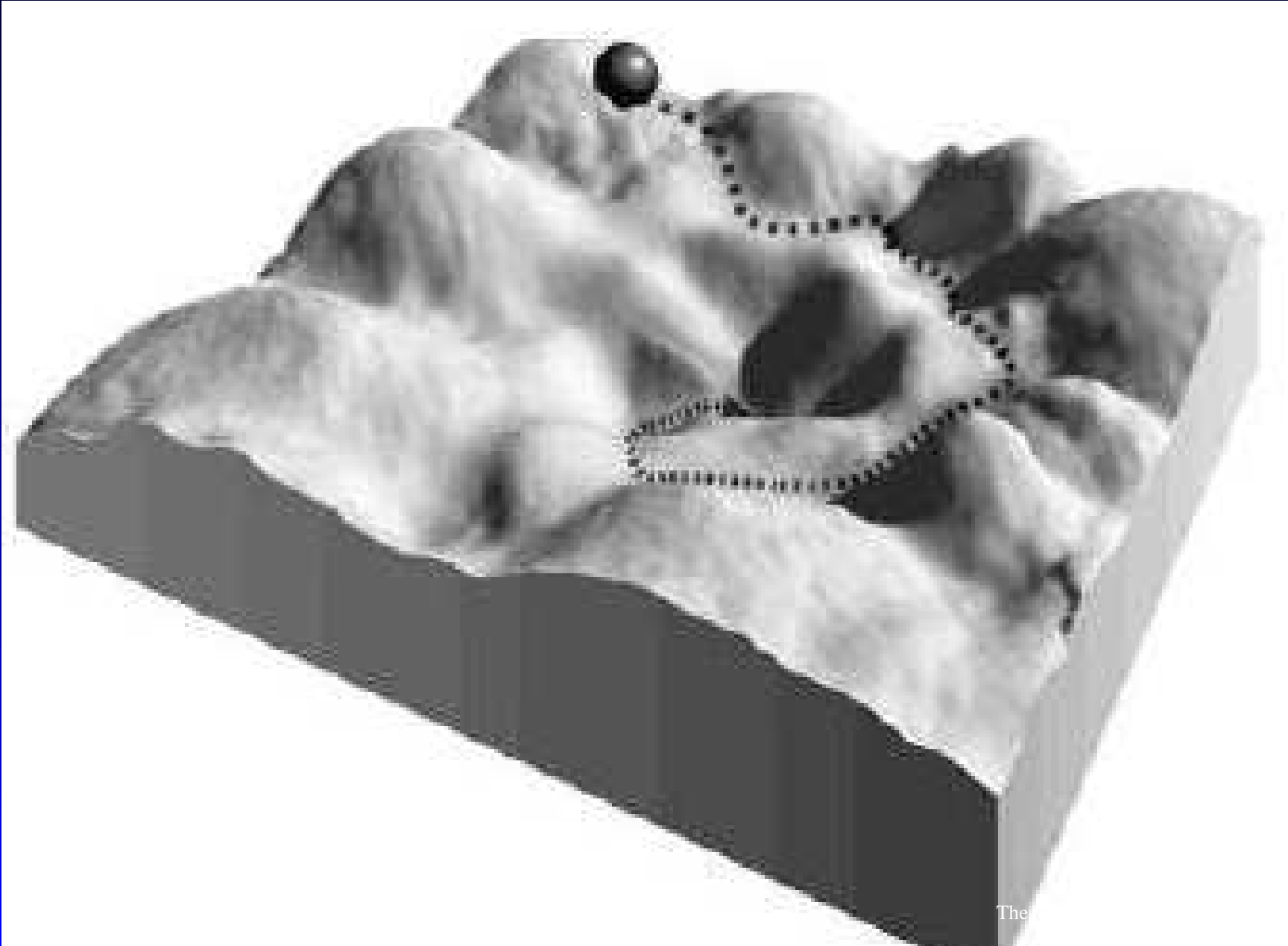
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- An intermediate step is to give a range of possible effective field theories that may arise in string theory.

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- An intermediate step is to give a range of possible effective field theories that may arise in string theory.
- Landscape: string theory has a vast metastable vacuum solutions (10^{500} ?)
Each corresponds to a low energy effective field theory (semi-classically consistent)

Landscape



Motivation

- Predictive power lost! Thus the landscape picture remains controversial both within and outside string theory.



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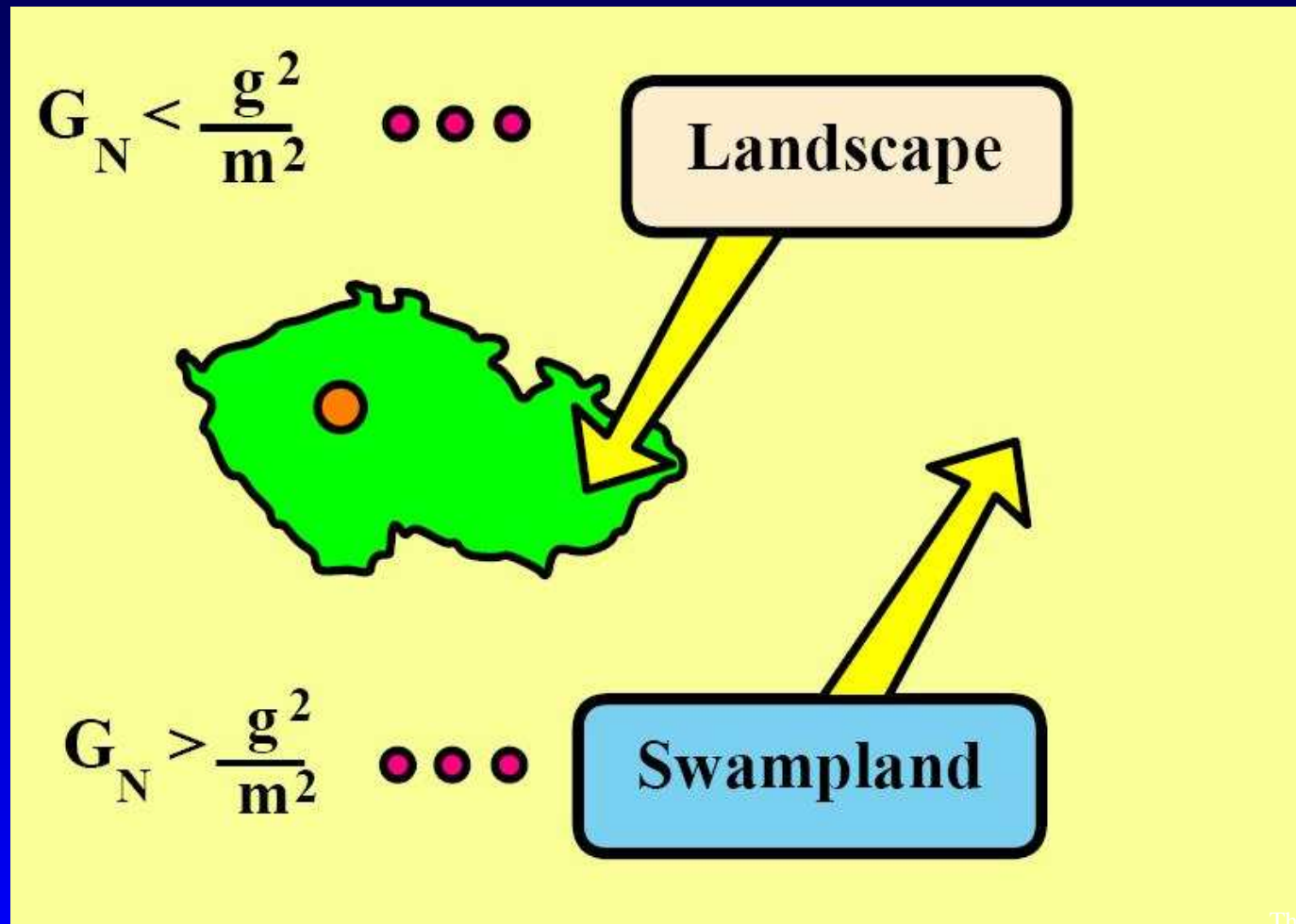
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Swampland

Consistent looking (at the semiclassical level) effective field theories which are actually inconsistent (at the full quantum level)



Some criteria

- To find consistent conditions for effective theories becomes important
- Key point:

A consistent quantum gravity theory coupled to matter

Note: general constraints for any consistent quantum gravity

Some conjectures

- C.Vafa
- N.Arkani-Hamed, L.Motl, A.Nicolis, C.Vafa
- S. Kachru, J. McGreevy, P. Svrcek
- A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, R. Rattazzi

Weak gravity conjecture

(Arkani-Hamed et.al)

- Conjecture: gravity is the weakest force in a certain sense
- Claim : there must be a light charged particle with a small mass $m_{el} \leq g_{el} M_{Pl}$
- Sharpened form:
$$\frac{M_{min}}{q M_{min}} \leq 1, \text{ for the lightest charged particle;}$$
- or: $\frac{M_q}{q_{min}} \leq 1, \text{ for the state with smallest mass/charge ratio}$

A surprising result

- Also true for magnetic monopole

$$m_{mag} \leq g_{mag} M_{Pl} \sim \frac{1}{g_{el}} M_{Pl}$$

- A monopole from $SU(2) \rightarrow U(1)$

- $m_{mag} \sim \frac{\Lambda}{g_{el}^2}$

- $\Lambda \leq g_{el} M_{pl}$

Evidence

- No global symmetries
 $g \rightarrow 0$ A gauge symmetry becomes a global one
- Black holes
remnants
a fundamental monopole is not a black hole:
$$M_{mon} \sim \frac{\Lambda}{g^2} < R_{mon} \sim \frac{1}{\Lambda}$$
$$\implies \Lambda \leq g_{el} M_{pl}$$
- String theory checks

Evidence

- no global symmetries

$$g \rightarrow 0$$

A gauge symmetry looks like a global one

Just the decoupling limit!

- black holes

remnants **Yet unclear!**

monopole is not a black hole:

$$M_{mon} \sim \frac{\Lambda}{g^2} < R_{mon} \sim \frac{1}{\Lambda}$$

$$\implies \Lambda \leq g_{el} M_{pl}$$

What about monopole becoming a black hole?

- string theory checks

Low dimensional evidence

(M.Li et.al)

- Aiming to find more convincing evidence
- $m \leq gM_{Pl} \leftrightarrow m \leq g/\sqrt{G}$
- The later form is unchanged under compactification
- e.g. on torus $g \rightarrow g/\sqrt{V}$, $\sqrt{G} \rightarrow \sqrt{G}/\sqrt{V}$

Low dimensional evidence

- Low dimension, simple physics
- If we expect an effective theory be descendent from a fundamental theory via symmetry breaking
it is natural to consider solitonic objects
- 3 examples with matter coupled to gravity

Soliton solutions in QFT

- Associated with symmetry breaking, non-perturbative
- Topology from the homotopy group of the gauge group
- Magnetic monopole $SU(2) \rightarrow U(1)$
- Nielsen-Olesen vortex $U(1) \rightarrow Z(2)$
- Instanton
- Kink

3D: Nielsen-Olsen vortex

- U(1) gauge field coupled to a complex scalar

- Action:

$$S = \int d^3x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - D_\mu \bar{\phi} D_\mu \phi - \frac{\lambda}{2} (\bar{\phi} \phi - F^2)^2 \right)$$

the scalar has a charge g ,

- $r \rightarrow \infty, \phi \rightarrow F e^{i\theta}, A_i \rightarrow -\epsilon_{ij} x_j / gr^2$

- Mass of a static vortex solution is

$$m = \frac{2\pi m_W^2}{g^2} C_1(\beta)$$

$m_W^2 = 2g^2 F^2$, $C_1(\beta)$ is a function of the dimensionless ratio $\beta = \lambda/g^2$ and parametrically is of order 1

3D: Nielsen-Olsen vortex

- 3D gravity

- a point like mass

$$ds^2 = -dt^2 + r^{-8Gm} [dr^2 + r^2 d\theta^2]$$

- deficit angle $8\pi Gm < 2\pi$

- otherwise,
the location of the particle $r = 0$ blows up to a
circle of infinite radius

3D: Nielsen-Olsen vortex

- $\implies m_W < g/\sqrt{G}$
- A charged particle satisfying the weak gravity conjecture found!
- Further, $m_W = \Lambda$, a constraint for the cutoff!
- Make no reference to black holes

3D instanton

- U(1) from SU(2)
- 3D instanton \leftrightarrow 4D monopole
- The action of an instanton is m_W/g^2
- The inequality implies $l_G < 1/m_W$
- The very condition to trust the field theory approximation
- Large gravitational corrections to the solution expected if $l_G > 1/m_W$

2D

- 2D kink solution?
- U(1) dipole coupled to dilaton gravity
$$S = \int d^2x \sqrt{-g} e^{-2\phi} (R + 4(\partial\phi)^2 - \frac{1}{2}F^2)$$
- Dipole solution in flat spacetime
- $F_{tx} = E = g(\theta(x) - \theta(x - l))$
- $E = 0$ outside $(0, 1)$ and $E = g$ inside $(0, 1)$

2D

- Energy of the dipole $g^2 l$
- Weak gravity conjecture: $g^2 l < g$
- or, $gl < 1$

2D

- lightcone coordinates, conformal gauge, E.O.M:
- $2\partial_+\phi\partial_-\phi - \partial_+\partial_-\phi - \frac{1}{2}e^{-2\rho}E^2 = 0,$
- $\partial_+\partial_-\rho - 2\partial_+\partial_-\phi + 2\partial_+\phi\partial_-\phi + \frac{1}{2}e^{-2\rho}E^2 = 0$
- $2\partial_+\rho\partial_+\phi - \partial_+^2\phi = 0$
- $2\partial_-\rho\partial_-\phi - \partial_-^2\phi = 0$

2D

- Outside $(0,1)$ $e^{2\rho} = \frac{1}{1+cx}$
 $c = 0$ to have a finite coupling at infinity
- Inside $(0,1)$ $e^{2\rho} = \frac{1}{1-4gx}$
- $4gl < 1$ to have regular solution inside the pair

2D

- $\int \sqrt{-g_{tt}} dt dx (g\delta(x) - g\delta(x - l))$
added to satisfy the continuous condition
- Not realistic
- A kink solution is currently under investigation.

With a positive cosmological constant

A magnetic charged black hole

$$ds^2 = -\left(1 - \frac{2Gm}{r} - \frac{r^2}{L^2}\right)dt^2 + \left(1 - \frac{2Gm}{r} - \frac{r^2}{L^2}\right)^{-1}dr^2 + r^2d\Omega^2$$

- Conditions $r_{bh} \leq \frac{1}{\Lambda} \leq L$ lead to

$$\Lambda^4 - \frac{g^2}{2G}\Lambda^2 + \frac{g^2}{GL^2} \leq 0 \quad L = \sqrt{3/(8\pi G\rho_V)}$$

- For a solution to exist $g \geq \frac{\sqrt{8G}}{L}$

With a positive cosmological constant

- One can obtain the same relation by starting with $m \leq gM_{pl}$ and
- $\frac{1}{\Lambda} < L$

With a positive cosmological constant

There is an interesting string interpretation of this result. We refer to [hep-th/0603127](https://arxiv.org/abs/hep-th/0603127) for more details.

Conclusions

- There are much more surprising results for us to learn about quantum gravity constraints on effective field theories.
- It is premature to say that the string landscape is a logical consequence of string theory.
- We are still in need of a more powerful selecting principle.

Thank you