Non-Gaussianity in KKLMMT model

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Received 21 September 2005; accepted 16 December 2005
Available online 27 December 2005
Editor: M. Cvetič

Abstract
We investigate the non-Gaussianity of the brane inflation which happens in the same throat in the framework of the generalized KKLMMT model. When we take the constraints from non-Gaussianity into account, various consequences are discussed including the bound on the string coupling, such as the string coupling is larger than 0.08 and the effective string scale on the brane is larger than 1.3 \times 10^{-4} M_p in KKLMMT model.

The cosmic microwave background (CMB) data from WMAP [1] strongly supports that inflation [2] should happen before the hot big bang. The first year results of WMAP also confirm the emerging standard model of cosmology, a flat Λ-dominated universe seeded by nearly scale-invariant adiabatic fluctuations. By using two statistics, Komatsu et al. [3] find that the data from WMAP consistent with Gaussian primordial fluctuations and establish limits, −58 < f_{NL} < 134, at 95% confidence, where f_{NL} is a non-linear coupling parameter which characterizes the amplitude of a quadratic term in the primordial potential.

One possible inflation model naturally set up in string theory is derived by the potential between the parallel dynamical brane and anti-brane, namely brane inflation [4–6]. However, the potential which governs the evolution of inflationary universe should be flat enough to satisfy the slow roll conditions. In the usual brane inflation, the tension of the brane (anti-brane) is too large and the attractive potential between them can be flat enough only when they separate far from each other. The authors of [6,8] pointed out that the distance between the brane and the anti-brane must be larger than the size of the extra-dimensional space if the slow roll inflation could happen in this scenario. This is called η problem. Kachru et al. in [7] successfully introduce some D3 branes in a warped geometry in type IIB superstring theory to break supersymmetry and uplift the AdS vacuum to a metastable de Sitter vacuum with lifetime long enough, but shorter than the recurrence time. If we take an extra pair of brane and anti-brane in this scenario, a more realistic slow roll inflation (KKLMMT inflation model) can be naturally set up [8]. A generalized KKLMMT inflation model has been also discussed in [9], where a possible conformal-like coupling between the scalar curvature and the inflaton is taken into account.

In general, f_{NL} contains an order one constant part and a momentum-dependent part, i.e., g_{NL}(\tilde{k}_1, \tilde{k}_2) (for example, see [10,11]). However, the present constraint on non-Gaussianity parameter from WMAP only provides a constraint on the constant part. Thus, the present constraint on the non-Gaussianity parameter cannot be used to constrain the inflation at all. However, recently the authors of [15] found that the tachyonic instability gives rise to non-Gaussianity parameter in the primordial perturbation. The tachyon usually appears in the brane inflation when the inflation ends and the distance between the brane and anti-brane becomes the same order as the string length. Therefore, the non-Gaussianity can provide a stringent constraints on the brane inflation model (see the second paper in [15]).

In this Letter, we estimate the non-Gaussianity parameter f_{NL} and investigate the constraints on the generalized KKLMMT model in detail. Combining the amplitude of the power spectrum and the constraints on the non-Gaussianity parameter, we can estimate the string coupling and the string scale on the brane. 

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parameter, we find there is a stringent constraints on the general-
ized KKLMMT model.
In KKLMMT inflation model, inflation is driven by the in-
teraction between a $D_3$ brane and $\bar{D}_3$ brane which are parallel and widely separated in five-dimensional AdS space. The $D_3$ brane is located at the bottom of the throat $A$ with warp fac-
tor $h_A$. The $D_3$ brane is mobile and slowly moves towards the $\bar{D}_3$ brane due to the attractive force between them. The distance between the brane and anti-brane plays the role of the inflaton field. We compactify the string theory on $AdS_5 \times X_5$, where $X_5$ is a five-dimensional Einstein manifold. The $AdS$ solution is given by the metric
\[ ds^2 = h^2(r)(-dt^2 + a^2(t)\,d\vec{x}^2) + h^{-2}(r)\,dr^2, \] (1)
with the warp factor
\[ h(r) = \frac{r}{R} = \exp\left(-\frac{2\pi K}{3 M g_s}\right), \] (2)
where $K$ and $M$ are the background NS–NS and R–R fluxes and $R$ is the curvature radius of the AdS throat [12],
\[ R^4 = \frac{27}{4}\pi g_s N_A \alpha'^2. \] (3)
Here $N = K M$ is just the number of the background $D_3$ charge. Since the curvature scales like $1/R^2$, $R, N_A$ should be large in order that the curvature is small and the supergravity analysis is reliable.
We set an $\bar{D}_3$ brane at the bottom of throat $A$ with coordi-
nate $r_0$. Its tension contribute a positive effective cosmological constant given by
\[ V_A = 2T_3h_A^4, \] (4)
where
\[ T_3 = \frac{1}{(2\pi)^2 g_s \alpha'^2} \] (5)
is the tension of $D_3$ brane. The force exerted by gravity and the
five-form field are of the same sign and add up, so we have a factor of 2 in Eq. (4). The factor of $h_A^4$ in (4) is due to a redshift in the curved geometry. Or from another point of view, we write down the string action in this background geometry as
\[ S = \frac{M_s^2}{2\pi} \int d^2z \, G_{AB} \partial X^A \bar{\partial} X^B \]
\[ \sim \frac{(M_s h_A)^2}{2\pi} \int d^2z \, g_{\mu\nu} \partial x^\mu \bar{\partial} x^\nu, \] (6)
where $M_s = 1/\sqrt{\alpha'}$ and $G_{AB}$ is the metric of the target spacetime, $X^A$ is the coordinate in the target spacetime and $x^\mu$ is the longitude coordinate on the brane in the metric (1). Thus for the observers living on the brane, the effective string scale should be $M_{\text{obs}} = M_s h_A$, which is different from the string scale in the bulk $M_s$. If the warp factor $h_A$ is smaller than one, the string seems lighter. This is the key point why the KKLMMT model can work. In addition, the attractive interaction also provides a potential
\[ V_{D_3 \bar{D}_3} = -\frac{27}{16\pi^2} \frac{T_3^2 h_A^8}{\phi^4}, \] (7)
where $\phi = \sqrt{T_3r}$ and $r$ is the coordinate corresponding to the position of the mobile $D_3$ brane. One must ensure that the compactification volume is stabilized in order to avoid the de-
compactification in KKLMMT model. A term coming from the Kähler potential and various interactions in the superpotential [8] and some possible D-terms [13] should also arise. The exact formulation of this potential is still not known. Usually, it can be written down as
\[ V_c = \frac{1}{2} \beta H^2 \phi^2, \] (8)
which induces a conformal like coupling between the scalar curvature and the inflaton, here $H$ is the Hubble parameter. Summing the above potentials up, we obtain the effective potential in the generalized KKLMMT model, given in [9] as follows:
\[ V = \frac{1}{2} \beta H^2 \phi^2 + 2h_A^4 T_3 \left(1 - \frac{1}{N_A} \phi_A^4 \right), \] (9)
where
\[ \phi_A^4 = (\sqrt{T_3r_0})^4 = \frac{27}{32\pi^2} N_A T_3 h_A^4. \] (10)
In Eq. (9), the parameter $\beta$ can be approximately regarded as a free parameter and the model reduces to KKLMMT model when $\beta = 0$. In [9], the authors offer the constraints on $\beta$. Roughly, the parameter $\beta$ should satisfy $0 \leq \beta \leq 1/7$ to 1/5.
This inflation model has been discussed in [9] and we di-
rectly quote the useful results. In this model, the slow roll pa-
rameter $\epsilon$ is always small. We need only focus on the slow roll parameter $\eta$ when we discuss the slow roll conditions. In the generalized KKLMMT model, $V_A$ dominated the evolution of the universe. Using Eq. (9), we have
\[ \eta = \frac{M_s^2 \frac{V''}{V}}{V} = \frac{\beta}{3} - \frac{20}{N_A} \frac{M_s^2 \phi_A^4}{\phi^6}, \] (11)
where $M_s$ is the reduced Planck mass in four-dimensional spacetime. The inflation ends when the slow roll conditions are broken down, here $\eta \sim -1$, which gives the final value of the inflaton field $\phi_{\text{end}}$ as
\[ \phi_{\text{end}} = \frac{20}{N_A} \frac{1}{1 + \beta/3} M_s^2 \phi_A^4. \] (12)
The number of e-folds can be expressed as
\[ N_e \simeq \frac{1}{M_s^2} \int_{\phi_{\text{end}}}^{\phi_f} \frac{V}{V} d\phi. \] (13)
So the value of $\phi$, namely, $\phi_{N_e}$ at the number of e-folding num-
ber $N_e$ before the end of inflation is
\[ \phi_{N_e}^{\text{end}} = \frac{24}{N_A} N_e M_s^2 \phi_A^4 m(\beta), \] (14)
where
\[ m(\beta) = \frac{(1 + 2\beta)e^{2\beta N_e} - (1 + \beta/3)}{2\beta(N_e + 5/6)(1 + \beta/3)}. \] (15)
Now the slow roll parameter can be expressed as
\[
\epsilon = \frac{M_p^2}{2} \left( \frac{V''}{V} \right)^2 = \frac{1}{18} \left( \frac{\phi_{\text{Ne}}}{M_p} \right)^2 \left( \beta + \frac{1}{2N_e m(\beta)} \right)^2,
\]
\[
\eta = \frac{\beta}{3} - \frac{5}{6} \frac{1}{N_e m(\beta)},
\]
where
\[
\frac{\phi_{\text{Ne}}}{M_p} = \left( \frac{3 \cdot 5^2}{25} \right)^{1/4} m^{1/6}(\beta) f(1/3) N_e^{-1/4} \delta_H^{1/2},
\]
here we use Eq. (14) and the amplitude of the primordial scalar power spectrum \(\delta_H\) is given by
\[
\delta_H = \frac{1}{\sqrt{75\pi} M_p^3} \frac{V^{3/2}}{V'},
\]
\[
= \left( \frac{2^{11}}{3 \cdot 56^4} \right)^{1/6} N_e^{5/6} \left( \frac{T_3 h_A^4}{M_p^4} \right)^{1/3} f^{-2/3}(\beta),
\]
where
\[
f(\beta) = \left( \frac{2\beta(N_e + 5/6)}{(1 + 2\beta)e^{2\beta N_e} - (1 + \beta/3)} \right)^{5/4} \times \left( \frac{(1 + 2\beta)^{5/2} e^{3\beta N_e}}{(1 + \beta/3)^{3/4}} \right)^{3\beta N_e}.
\]
The cosmological observations [14] show that \(\delta_H \sim 1.9 \times 10^{-5}\) at \(N_e \sim 55\). In the limit with \(\beta \to 0\), we find \(f(\beta) \to 1\) and the results are just the same as the KKLMMT model. Using Eq. (18), we obtain
\[
g(\beta) = \left( \frac{2\beta(N_e + 5/6)}{(1 + 2\beta)e^{2\beta N_e} - (1 + \beta/3)} \right)^{5/4} \times \left( \frac{(1 + 2\beta)^{5/2} e^{3\beta N_e}}{(1 + \beta/3)^{3/4}} \right)^{3\beta N_e}.
\]
This formula offers a stringent constraint on the effective string scale on the brane and the string coupling from the cosmological observations. If we require that the inflation happens in the throat, the inflaton should satisfies
\[
\phi_A \leq \phi_{\text{end}} < \phi_{\text{Ne}} \leq \phi_R,
\]
here \(\phi_R = \sqrt{T_3 R}\) and the e-folding number \(N_e\) should be roughly larger than 55 in order to solve the flat and horizon problem in hot big bang cosmology. Using Eqs. (14), (20) and (21), we obtain an constraint on the tension of the \(D_3\) brane as
\[
\frac{T_3}{M_p^4} \geq \frac{25 \pi^2}{N_e N_A} m^{2/3}(\beta) f^{4/3}(\beta) \delta_H^2.
\]
On the other hand, the reduced Planck scale in the four-dimensional spacetime is given by
\[
M_p^2 = \frac{2 L^6}{(2\pi)^7 \alpha'/4 g_s^2},
\]
where \(L\) is the characteristic size of the compactified space. The ratio between the \(D_3\) brane tension and the Planck energy density can be expressed as
\[
\frac{T_3}{M_p^4} = \frac{(2\pi)^{11}}{4} g_s^3 \left( \frac{l_s}{L} \right)^{12},
\]
here \(l_s = 1/M_s\). In order to make the concept of the geometry reliable, we require that the size of the compactified space should be larger than the length of the string, i.e., \(l_s \leq L\), which means that
\[
\frac{T_3}{M_p^4} \leq \frac{(2\pi)^{11}}{4} g_s^3.
\]
Combining Eqs. (22) and (25), we find a constraint on the background charge \(N_A\) and the string coupling \(g_s\) as follows:
\[
N_A \geq \frac{25}{(2\pi)^9 N_e g_s^3} m^{2/3}(\beta) f^{4/3}(\beta) \delta_H^2.
\]
This inequality can be easily satisfied if the string coupling is not too small. Here we also require that the inflation should end before the distance between \(D_3\) brane and \(D_3\) brane reaches string length \(\sqrt{a'}\), which imposes a new constraints on the string coupling \(g_s\) and the background charge \(N_A\) as (see [9])
\[
\ln N_A + 4 \left( \frac{4}{2\pi g_s N_A} \right)^{1/4} \leq \frac{1}{3} \ln \left( \frac{2^9 \cdot 5^2 \pi^2}{3^3} \right) \frac{1}{(1 + \beta/3)^2} g(\beta).
\]
This inequality, we can find that there is an up bound for the background charge, e.g., \(N_A \leq 6.9 \times 10^9\) for \(\beta = 0\) and \(N_A \leq 1.4 \times 10^8\) for \(\beta = 0.1\). Combining Eq. (26), we can find a low bound for the string coupling \(g_s\), e.g., \(g_s \geq 1.2 \times 10^{-8}\) for \(\beta = 0\) and \(g_s \geq 3.6 \times 10^{-6}\) for \(\beta = 0.1\).

In KKLMMT model, the distance between the \(D_3\) brane becomes roughly the same order as the string length and the tachyon appears when inflation ends. In [15], the authors point out that the non-Gaussianity arise due to the instability of the tachyon and they estimate the non-Gaussianity in the brane inflation model. Here we estimate the non-Gaussianity in the generalized KKLMMT model. We propose that the action for the tachyon in KKLMMT model can be expressed as (the superstring tachyon for the brane and anti-brane pair, for example, see [16])
\[
S_3 = -2 T_3 h_A^4 \Delta \times \int d^4 x \sqrt{-g} \left( \frac{\alpha'}{2h_A^4} e^{-|T|^2/4} \partial_{\mu} T^4 \partial^\mu T + e^{-|T|^2/4} \right).
\]
The factor \(h_A^4\) appears because of the geometry effect in the throat, we should use the effective string scale instead of the string scale in the bulk. In the \(D\bar{D}\) system, tachyon is a complex filed. Here we may consider the simplest case where the imaginary part of \(T\) is essentially frozen and only the real part roll down (see [18] for the argument). We redefine the tachyon field as
\[
\varphi = \int_0^T \sqrt{\frac{2T_3 h_A^4}{M_{\text{obs}}^2}} e^{-T'^2/8} dT' = 2 \sqrt{\frac{\pi T_3 h_A^4}{M_{\text{obs}}^2}} \text{erf} \left( \frac{T}{2\sqrt{2}} \right).
\]
The transition $T = 0 \to \infty$ corresponds to $\varphi = 0 \to 2\sqrt{\pi T_3 h_A^4/M_{\text{obs}}^2}$. Now the mass square of the tachyon field $\varphi$ becomes

$$M^2_{\varphi} = -\frac{M^2_{\text{obs}}}{2} \left(1 - \frac{T^2}{4}\right).$$

(30)

When $T > 2$, $\varphi$ will not be a tachyon field any more. So, $\varphi$ can be taken as a tachyon field only when $0 \leqslant \varphi \leqslant \varphi_m$ with

$$\varphi_m = \frac{1}{\sqrt{2}} \sqrt{\frac{\pi T_3 h_A^4}{M_{\text{obs}}^2}} \text{erf} \left(\frac{1}{\sqrt{2}}\right).$$

As a result, long wavelength quantum fluctuations of the tachyon field $\varphi$ with momenta smaller than $M_{\varphi}$ grow exponentially. As given in [15], the fluctuation of the tachyon can be expressed as

$$\langle \delta \varphi^2 \rangle = \int_0^{M_{\text{obs}}/\sqrt{2}} k \, dk \frac{e^{-2T \sqrt{\ln(1/\sqrt{B_{gs}})}}}{4\pi^2 e^{-2k^2}} = e^{\sqrt{2}M_{\text{obs}}/\sqrt{2}} \text{erf} \left(\frac{1}{\sqrt{2}}\right).$$

(31)

The growth of the tachyon fluctuation continues until $\sqrt{\langle \delta \varphi^2 \rangle}$ reaches the value $\varphi_m$, since at $\varphi \sim \varphi_m$ the curvature of the effective potential vanishes and instead of exponential growth on has the usual oscillations of all the modes. We can estimate that the time span is $t_e \sim (\sqrt{2}/M_{\text{obs}}) \ln(1/\sqrt{B_{gs}})$, where $B = (8\pi^2/1/\sqrt{2}))^{-1}$. During this period, the number density of the tachyon quanta is given by [17]

$$n_k \sim e^{\sqrt{2}M_{\text{obs}}t_0} \sim \frac{1}{B_{gs}}.$$  

(32)

Thus the number density of the tachyon quanta in x-space is

$$n_\varphi = \int_0^{M_{\text{obs}}/\sqrt{2}} \frac{d^3k}{(2\pi)^3} n_k \sim \frac{1}{(2\pi)^3 B_{gs}^2} \left(\frac{M_{\text{obs}}}{\sqrt{2}}\right)^3.$$

(33)

Just the same as [15], we assume that the VEV of the inflaton is vanishing, $\langle \phi \rangle = 0$, when the tachyon starts rolling. The tachyon field and the inflaton field can be divided into the background, the first order and the second order perturbation as

$$\varphi = \varphi_0(\chi) + \delta^{(1)}\varphi(\chi, \bar{\chi}) + \frac{1}{2} \delta^{(2)}\varphi(\chi, \bar{\chi}),$$

$$\phi = \delta^{(1)}\phi(\chi, \bar{\chi}) + \frac{1}{2} \delta^{(2)}\phi(\chi, \bar{\chi}),$$

(34)

where $\chi$ is conformal time. The relevant part of the metric perturbations is

$$g_{00} = -a(t)^2 (1 + 2\psi^{(1)} + \psi^{(2)}).$$

(35)

Then the first order and second order perturbation equations can be written as (see [15] for detail form)

$$\left(\psi^{(1)}(\chi, \bar{\chi})\right)^{\prime\prime} - 2A \left(\psi^{(1)}(\chi, \bar{\chi})\right)^{\prime} \sim 0,$$

(36)

$$\left(\psi^{(2)}(\chi, \bar{\chi})\right)^{\prime\prime} - 2A \left(\psi^{(2)}(\chi, \bar{\chi})\right)^{\prime} \sim - \frac{1}{M_p^2} \left(2\delta^{(1)}\varphi\right)^2 + 8\psi_0^2 \left(\psi^{(1)}\right)^2$$

$$- a^2 V_{\varphi\varphi} \left(\delta^{(1)}\varphi\right)^2 - 8\psi_0 \psi^{(1)} \delta^{(1)}\varphi^\prime,$$

(37)

where $A = \varphi_0^{(2)}/\varphi_0^{(1)}$ and the primes denote derivative with respect to the conformal time $\chi$. We assume that the tachyon modes grow within a time interval much smaller than the Hubble time and the expansion of the universe can be neglected, so that we can set $\chi = t$, $a = 1$ and $A = \dot{\varphi}_0/\varphi_0$ which can be taken as a constant. There are two solution with the first order metric perturbations: $\psi^{(1)}$ is a constant and $\psi^{(1)} \propto e^{2\beta t}$. We can consider these two cases separately.

If the constant solution dominating, we can take $\psi^{(1)} \sim 10^{-5}$ by using the observed temperature anisotropies. Now the total energy density stored in $\varphi$ is

$$\rho_\varphi \sim \frac{1}{2} \left(\delta^{(1)}\varphi\right)^2 \sim n_\varphi \frac{M_{\text{obs}}}{\sqrt{2}} \sim \frac{M_{\text{obs}}^4}{32\pi^3 B_{gs}}.$$

(38)

Now Eq. (37) becomes

$$\dot{\psi}^{(2)} \sim -\frac{2}{M_p^2} \left(\delta^{(1)}\varphi\right)^2 \sim - \frac{1}{8\pi^3 M_p^2} \frac{1}{B_{gs}}.$$  

(39)

here we use Eq. (38). The second order metric perturbation

$$\psi^{(2)} \sim - \frac{1}{8\pi^3 M_p^2} \frac{1}{B_{gs}} \left(M_{\text{obs}}/M_p\right)^2 \ln^2 \left(\frac{1}{\sqrt{B_{gs}}}\right).$$

(40)

Thus the standard non-Gaussianity parameter $f_{\text{NL}}$ is given by

$$f_{\text{NL}} = - f_{\text{NL}} + \frac{11}{6} \sim \frac{\psi^{(2)}}{\psi^{(1)}} + \frac{11}{6}$$

$$\sim \frac{11}{6} + \frac{10^{10}}{8\pi^3} \frac{1}{B_{gs}} \left(M_{\text{obs}}/M_p\right)^2 \ln^2 \left(\frac{1}{\sqrt{B_{gs}}}\right).$$

(41)

Based on Eq. (41), there is a stringent constraint on the string coupling $g_s$ and the effective string scale on the brane $M_{\text{obs}}$ from WMAP (see Fig. 1).

The region below the red line in Fig. 1 is allowed by the present constraints from WMAP. Here we set $N_r \sim 55$, $\delta \eta = 1.9 \times 10^{-5}$, for each $\beta$. Combining the constraints of the amplitude of the primordial power spectrum (20), we find that the non-Gaussianity can give us a stringent constraints on the string coupling and the effective string scale on the brane:

(1) for $\beta = 0$, the string coupling $g_s \geqslant 10^{-1.1} \pm 0.08$ and $M_{\text{obs}}/M_p \geqslant 1.3 \times 10^{-4}$. The constraints on the cosmic F-string and D-string will be:

$$G_{\mu_F} = \frac{1}{8\pi} \left(M_{\text{obs}}/M_p\right)^2 \geqslant 6.7 \times 10^{-10},$$

$$G_{\mu_D} = GT_3 h_A^2 = \left(\frac{1}{32\pi g_s} \frac{T_3 h_A^4}{M_p^4}\right)^{1/2} \leqslant 1.3 \times 10^{-9},$$

(2) for $\beta = 0.1$, the string coupling $10^{0.35} \sim 2.2 \leqslant g_s \leqslant 10^{0.87} \sim 7.4$ and $3.0 \times 10^{-4} \leqslant M_{\text{obs}}/M_p \leqslant 4.0 \times 10^{-4}$. The constraints on the cosmic F-string and D-string are:

$$3.6 \times 10^{-9} \leqslant G_{\mu_F} \leqslant 6.4 \times 10^{-9} \text{ and } 4.6 \times 10^{-8} \leqslant G_{\mu_D} \leqslant 7.4 \times 10^{-8}.$$
Substituting this equation into (37) and solving it, we obtain the perturbation is dominated by the exponential solution $G\mu$ on the string tension with strings to the CMB power spectrum provides an upper limit on the string tension with $G\mu = \frac{1}{8\pi} (M_{\text{obs}}/M_p)^2 < 10^{-6}$ [19].

Therefore, for $\beta = 0.1$, the case with string coupling larger than $10^{5.8}$ should be ruled out. If we also require that $g_s \lesssim 1$ should be allowed, the parameter $\beta$ should satisfy $\beta \lesssim 0.03$.

The other extreme case is when the first order metric perturbation is dominated by the exponential solution $\psi^{(1)} \propto e^{2A_t}$. This can happen if the initial fluctuation of the tachyon field is large enough to overcome the constant solution for the first order perturbation in the metric. In this case, the first order perturbation of the tachyon field can be given by

$$\delta^{(1)} \psi = \frac{2M_p^2}{\phi_0^2} (\psi^{(1)} + H\psi^{(1)}) \sim 4A \frac{M_p^2}{\phi_0} e^{2A_t}. \quad (42)$$

Substituting this equation into (37) and solving it, we obtain the relevant second order perturbation as

$$\psi^{(2)} \sim (8 + \frac{2M_p^2 V''(\phi)}{\phi_0^2} - \frac{16M_p^2}{\phi_0^2} - \frac{\phi_0^4}{M_p^2 \phi_0^2}) e^{4A_t}. \quad (43)$$

Thus, the non-Gaussianity parameter can be given by

$$f_{\text{NL}} = f_{\text{NL}}^{\psi} + \frac{11}{6} \sim \left(\frac{\psi^{(2)}}{(\psi^{(1)})^2}\right)^2 + \frac{11}{6},$$

$$\sim \frac{37}{6} + 16C g_s \left(\frac{M_p}{M_{\text{obs}}}\right)^2 + \frac{1}{C g_s} \left(\frac{M_{\text{obs}}}{M_p}\right)^2,$$

$$+ 2C g_s \left(\frac{M_p}{M_{\text{obs}}}\right)^2 \ln^2 \left(\frac{1}{\sqrt{B g_s}}\right),$$

(44)

with $C = 2\pi^2/\text{erf}^2(1/\sqrt{2})$. The constraint on the string coupling and the effective string scale on the brane is shown in Fig. 2.

**Fig. 2.** $x = \log_{10} g_s$ and $y = \log_{10}(M_{\text{obs}}/M_p)$. The red line corresponds to $f_{\text{NL}} = 134$ and the region below the red line is allowed by WMAP. The green and blue lines come from the constraints of the amplitude of the primordial scalar power spectrum (Eq. (20)) with different parameter $\beta$. Here the green line corresponds to $\beta = 0$ and the blue line to $\beta = 0.1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the Letter.)

From Fig. 2, the string coupling must be smaller than the limit in order for the generalized KKLMMT model to work, which we have obtained before. Thus if the first order metric perturbation is dominated by the exponentially growing solution, the generalized KKLMMT model must be ruled by the constraint from the non-Gaussianity.

In summary, we estimate the non-Gaussianity due to the tachyon instability in the KKLMMT model. If there is a large first order metric perturbation due to the fluctuation of tachyon, the generalized KKLMMT inflation model has been ruled out by the present non-Gaussianity constraint from WMAP. When the first order perturbation of metric is not amplified due to the rolling tachyon, there is still some stringent constraints on the string coupling and the effective string scale on the brane in the generalized KKLMMT model. These constraints provide some bounds for the cosmic F-string and D-string tension. We expect that the cosmological observations in the future, including WMAP, Planck and LIGO, will offer better opportunity for testing the KKLMMT inflation model.

**Acknowledgements**

We thank Y.F. Chen, M. Li, J.X. Lu and H. Tye for useful discussions. The research of Huang is supported by a grant from NSFC, a grant from China Postdoctoral Science Foundation and a grant from K.C. Wang Postdoctoral Foundation.

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