Large non-Gaussianity implication for curvaton scenario

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Abstract

We argue that the typical energy density of a light scalar field should not be less than $H^4$ in the inflationary Universe. This requirement implies that the non-Gaussianity parameter $f_{NL}$ is typically bounded by the tensor–scalar ratio $r$ from above, namely $f_{NL} \lesssim 518 \cdot r^4$. If $f_{NL} = 10^2$, inflation occurred around the GUT scale.

1. Introduction

Inflation model [1–3] provides an elegant mechanism to solve the horizon, flatness and primordial monopole problem due to a quasi-exponential expansion of the universe before the hot big bang. The temperature anisotropies in cosmic microwave background radiation (CMBR) and the large-scale structure of the Universe are seeded by the primordial quantum fluctuations during inflation [4,5]. Since the density perturbation is roughly $10^{-5}$, it is good enough to apply the linear perturbation theory to calculate the quantum fluctuations during inflationary epoch. Within this approach, the Fourier components of fluctuations are uncorrelated and their distribution is Gaussian. That is why the non-Gaussianity from the simplest inflation models is very small ($|f_{NL}| < 1$). For useful discussions on non-Gaussianity see [6–11], and for a nice review see [12].

The non-Gaussian perturbation is governed by the $n$-point correlation function of the curvature perturbation

$$\langle \Phi(k_1)\Phi(k_2)\cdots\Phi(k_n) \rangle = (2\pi)^3 \delta^3 \left( \sum_{i=1}^{n} k_i \right) F_{NL}(k_1,k_2,\ldots,k_n),$$

(1)

where $\Phi(k)$ is the Fourier mode of Bardeen’s curvature perturbation. The leading non-Gaussian features are known as the bispectrum (three-point function) and trispectrum (four-point function), with their sizes conventionally denoted as $f_{NL}$ and $g_{NL}$, respectively. The non-linearity parameter $f_{NL}$ defined in [13] takes the form

$$\Phi(x) = \Phi_L(x) + f_{NL} \left[ \Phi^2_L(x) - \langle \Phi^2_L(x) \rangle \right],$$

(2)

where $\Phi_L$ denotes the linear Gaussian part of the perturbation in real space. The non-Gaussianity parameter $f_{NL}$ characterizes the amplitude of the primordial non-Gaussian perturbations. The value of $\Phi_L$ is roughly $10^{-5}$. If $f_{NL} = 10^2$, the distribution of $\Phi$ is still consistent with a Gaussian distribution to 0.1%. It is not easy to detect a small non-Gaussianity in the experiments. The shape of non-Gaussianity is also very important. If $F(k_1,k_2,k_3)$ is large for the configurations in which $k_1 \ll k_2, k_3$, it is called local, “squeezed” type; if $F(k_1,k_2,k_3)$ is large for the configurations in which $k_1 \sim k_2 \sim k_3$, it is called non-local, “equilateral” type.

Recently a large primordial local-type non-Gaussianity was reported to be marginally detected from the data of WMAP3 in [14]:

$$27 < f_{NL} < 147$$

(3)

at 95% C.L., with a central value of $f_{NL} = 87$. See also [15]. In [16] WMAP group recently reported their new result as

$$-9 < f_{NL} < 111.$$  

(4)

If a large local-type non-Gaussianity is confirmed by the future cosmological observations and data analysis at high confidence level, it strongly implies that the physics of the early Universe is more complicated than the simple single-field slow-roll inflation.

There are several mechanisms to generate a large local-type non-Gaussianity:

1. a non-linear relation between inflaton and curvature perturbations [9];
2. curvaton scenario [17–22];
3. Ekpyrotic model [23–26].

In this Letter we focus on curvaton scenario. As we know, there might be many light scalar fields in supersymmetric theory or string theory. These light scalar fields which are subdominant during inflation are called curvaton. The energy density during inflation is still dominated by the potential of inflaton. In the usual inflation model the fluctuations of inflaton dominate the curvature...
perturbations. The energy density and the perturbations caused by these light scalar fields can be ignored during inflation. However, it is possible that the fluctuation of curvaton becomes relevant and causes a large local-type non-Gaussianity when its energy density is a significant fraction of the total energy after the end of inflation. In curvaton scenario, the perturbations from the inflaton field are considered to be negligible. A large local-type non-Gaussianity may shed light on these light scalar fields.

The non-Gaussianity produced by curvaton is in inverse proportion to the fraction of curvaton energy density in the energy budget at the epoch of curvaton decay. The non-Gaussianity increases as the curvaton energy density decreases. However, the energy density for a homogeneous scalar field in the quasi-de Sitter space is typically given by \( H^2 \). Therefore, the non-Gaussianity has an upper bound

\[ f_{NL} \leq 518 \cdot r^2, \tag{5} \]

where \( r \) is the tensor–scalar ratio. Present bound on the tensor–scalar ratio [16] is \( r < 0.20 \) and then \( f_{NL} \leq 346 \).

Our Letter is organized as follows. We review curvaton scenario and the curvature perturbation generated by curvaton in Section 2. In Section 3, we argue that the energy density of curvaton is typically estimated as \( H^2 \) during inflation and then the non-Gaussianity parameter \( f_{NL} \) is bounded by the tensor–scalar ratio from above. In Section 4, we point out the challenge for getting a red-tilted primordial power spectrum in curvaton scenario. Section 5 contains some discussions including how to distinguish the curvaton scenario from Ekpyrotic scenario.

### 2. Non-Gaussianity in curvaton scenario

In curvaton scenario, the dynamics of inflation in the early Universe is dominated by inflaton and the energy density of curvaton is negligible during inflation. The Lagrangian of a curvaton field \( \sigma \) in an unperturbed Friedmann–Robertson–Walker (FRW) spacetime is given by

\[ \mathcal{L}_\sigma = \frac{1}{2} \dot{\sigma}^2 - \frac{1}{2}(\nabla \dot{\sigma})^2 - V(\sigma), \tag{6} \]

where the subscript "p" denotes the derivative with respect to the physical coordinate. For simplicity, we consider a potential of curvaton as follows

\[ V(\sigma) = \frac{1}{2} m^2 \sigma^2. \tag{7} \]

The curvaton field is supposed to be an almost free field with a small effective mass compared to the Hubble scale \( H \) during inflation. Effectively we define a parameter

\[ \eta_{\sigma \sigma} = \frac{1}{3H^2} \frac{d^2V(\sigma)}{d\sigma^2} \tag{8} \]

which is much smaller than 1. The equation of motion for a homogeneous curvaton field takes the form

\[ \ddot{\sigma} + 3H\dot{\sigma} = -m^2 \sigma. \tag{9} \]

Since \( m \ll H \), the friction term \( 3H\dot{\sigma} \) is dominant and the value of curvaton is roughly a constant at the inflationary epoch. It is denoted as \( \sigma = \sigma_\ast \).

The amplitude of the quantum fluctuation of curvaton field in a quasi-de Sitter space is given by

\[ \delta \sigma = \frac{H_\ast}{2\pi}. \tag{10} \]

The spectrum of the fractional perturbations caused by curvaton [19] is

\[ P_{\delta \sigma / \sigma} = \frac{\delta \sigma}{\sigma} = \frac{1}{2\pi} \frac{H_\ast}{\sigma_\ast}. \tag{11} \]

where * denotes the epoch of horizon exits \( k_\ast = a_\ast H_\ast \) during inflation. The quantum fluctuation of curvaton field is frozen at horizon exit to a classical perturbation with a flat spectrum. Since the curvaton energy density is subdominant in this epoch, its fluctuations are initially taken as isocurvature/entropy fluctuations. The curvaton perturbation is sourced on large scales [12].

After the end of inflation the inflaton energy density is converted into radiations and then the Hubble parameter decreases as \( a^{-2} \). The curvaton field will remain approximately constant \( \sigma_\ast \) until \( H \sim m \). At this epoch the curvaton starts to oscillate harmonically about \( \sigma = 0 \). During the stage of oscillating the curvaton energy density goes like \( \rho_\sigma \propto a^{-3} \) which increases with respect to the energy density of radiation \( \rho_R \propto a^{-4} \). When the Hubble parameter goes to the same order of the curvaton decay rate \( \Gamma \), the curvaton energy is converted into radiations. Finally, before primordial nucleosynthesis, the curvaton field is supposed to completely decay into thermalized radiation, thus the perturbations in the curvaton field are converted into curvature perturbations and become the final adiabatic perturbations which seed the matter and radiation density fluctuations observed in the Universe. We illustrate the evolution of one curvaton perturbation mode and Hubble radius in Fig. 1.

After the curvaton decay the total curvature perturbation will remain constant on superhorizon scales at a value which is fixed at the epoch of curvaton decay. The sudden-decay approximation is exact if the curvaton completely dominates the energy density before it decays. Going beyond the sudden-decay approximation, a numerical calculation in [27] implies that the spectrum of the curvature perturbations caused by curvaton is proportional to the fraction of curvaton energy density in the energy budget at the epoch of curvaton decay

\[ \Omega_{\sigma, D} = \left( \rho_\sigma / \rho_{\text{flat}} \right)_D. \tag{12} \]

A precise form of the spectrum in curvaton scenario is given in [21] as follows

\[ P_{\delta \sigma / \sigma}^D = \frac{2}{3} \Omega_{\sigma, D} \Omega_{\sigma, D} \frac{1}{3\pi} \Omega_{\sigma, D} \frac{H_\ast}{\sigma_\ast}. \tag{13} \]

In curvaton scenario the curvature perturbation is dominated by curvaton perturbation and then the spectral index in the curvaton model takes the form

\[ n_s = 1 + 2\eta_{\sigma \sigma} - 2\epsilon_H, \tag{14} \]

where \( \epsilon_H \) is the slow-roll parameter which is defined as

\[ \epsilon_H \equiv - \frac{H_\ast}{\dot{H}_\ast}. \tag{15} \]
In [21, 28] the non-Gaussianity parameters corresponding to bispectrum\(^1\) and trispectrum are respectively given by
\[
f_{\text{NL}} \simeq \frac{5}{4\Omega_{\sigma, D}}, \tag{16}\]
and
\[
\tau_{\text{NL}} = \frac{36}{25} f_{\text{NL}}. \tag{17}\]

In sudden-decay approximation, Eq. (16) should be replaced by \(5/(3\Omega_{\sigma, D})\). A large non-Gaussianity is obtained if \(\Omega_{\sigma, D} \ll 1\). In this case the Hubble parameter is always dominated by the radiation energy density before curvaton decays. Assume the scale factor is \(a(t_D) = 1\) at the moment when curvaton start to oscillate. The energy density of curvaton and radiation are \(\rho_\sigma(t_D) = \frac{1}{2}m^2\sigma_\sigma^2\) and \(\rho_R(t_D) = 3M_P^2n^2\), respectively. Since the radiation energy density goes like \(a^{-4}\), \(\rho_R(t_D) \simeq 3M_P^2f^2 \simeq \rho_R(t_0)a^{-4}(t_D)\) and thus \(a(t_D) \simeq (m/T)^{\frac{1}{2}}\). Therefore
\[
\Omega_{\sigma, D} \simeq \frac{\rho_\sigma(t_D)}{\rho_R(t_D)} = \frac{\sigma_\sigma^2}{6M_P^2}\left(\frac{m}{T}\right)^{\frac{1}{2}}. \tag{18}\]

This result is also given in [19]. Combing (16) with (18), we get
\[
f_{\text{NL}} = \frac{15}{2} \frac{M_P^2}{\sigma_\sigma^2}\left(\frac{T}{m}\right)^{\frac{1}{2}}. \tag{19}\]

Since \(T < m\), a large non-Gaussianity is obtained only when \(\sigma_\sigma \ll M_p\). It is reasonable to require that the VEV of curvaton is less than Planck scale and a large \(f_{\text{NL}}\) is expected.

The WMAP normalization [16] is
\[
P_T = 2.457 \times 10^{-9}. \tag{20}\]

Using Eqs. (13) and (16), the value of curvaton during inflation is related to the Hubble parameter by
\[
\sigma_\sigma = 2.68 \times 10^9 \frac{1}{f_{\text{NL}}} H_s. \tag{21}\]

The non-Gaussianity parameter \(f_{\text{NL}}\) should be smaller than \(2.68 \times 10^9\); otherwise, the quantum fluctuation of curvaton is greater than its VEV and the previous semi-classical description breaks down. Substituting (21) into (19), we find
\[
f_{\text{NL}} \simeq 10^6 \frac{H_s^2}{M_P^2}\left(\frac{m}{T}\right)^{\frac{1}{2}}. \tag{22}\]

The limit of the primordial gravitational wave perturbation implies that \(H_s < 10^{-3} M_p\), and then the mass of curvaton must be much larger than its decay rate for getting a large non-Gaussianity.

Tensor/gravitational wave perturbation encodes a very important information about inflation. Its amplitude only depends on inflation scale
\[
P_T = \frac{H_s^2}{M_P^2}\left(\frac{m}{T}\right)^{\frac{1}{2}}. \tag{23}\]

For convenience, we define a new parameter named the tensor–scalar ratio \(r\) as
\[
r = \frac{P_T}{P_\gamma}. \tag{24}\]

If the curvature perturbation is dominated by inflaton field we have \(r = 16e_H\) for slow-roll inflation. In curvaton scenario the density perturbation is dominated by curvaton and thus
\[
r < 16e_H. \tag{25}\]

According to (23), the Hubble parameter during inflation is related to the tensor–scalar ratio by
\[
H_s = 10^{-4}r^{1/2} M_p = 2.44 \times 10^{14}r^{1/2} \text{ GeV}. \tag{26}\]

Now Eq. (22) becomes
\[
f_{\text{NL}} = 10^{-2}r^{1/2}\left(\frac{m}{T}\right)^{1/2}. \tag{27}\]

WMAP gives an upper bound on the tensor–scalar ratio \(r < 0.20\) [16] and we need \(m/T > 2.5 \times 10^8\) for \(f_{\text{NL}} = 10^2\). The bound on the tensor perturbation also provides an upper bound on the inflation scale
\[
H_s < 1.09 \times 10^{14} \text{ GeV}. \tag{28}\]

We will see that the non-Gaussianity in curvaton scenario will give a lower bound on \(H_s\) in the next section.

3. Upper bound on non-Gaussianity \(f_{\text{NL}}\) in curvaton scenario

The curvaton mass and its decay rate are fixed if the field theory for the whole system is given. Since the curvaton does not move during inflation, we cannot use its dynamics to determine the value of \(\sigma_\sigma\). According to Eq. (19), \(f_{\text{NL}}\) is large if \(\sigma_\sigma\) is much smaller than the Planck scale. In the literatures, \(\sigma_\sigma\) is not fixed by the theory, but rather represents an additional free parameter, and then the non-Gaussianity parameter \(f_{\text{NL}}\) can be arbitrarily large.

This treatment is reliable at the classical level. However we will see that the quantum fluctuations of curvaton will significantly affect the value of curvaton during inflation and an upper bound on the typical value of \(f_{\text{NL}}\) is obtained for the curvaton model.

In the inflationary Universe the quantum fluctuations of the curvaton field freeze out with amplitude \(H/(2\pi)\) at the wavelength \(H^{-1}\). In [29] a gradient energy density due to these fluctuations is estimated
\[
\langle \nabla_\mu \sigma \nabla_\nu \sigma \rangle \sim H_s^4. \tag{29}\]

It is also obtained through calculating the contribution of these quantum fluctuations to the average value of the energy–momentum tensor. The gradient energy density characterizes the inhomogeneity of the curvaton and should be smaller than its homogeneous energy density \(\frac{1}{2}m^2\sigma_\sigma^2 \geq H_s^4\). Requiring \(m^2\sigma_\sigma^2 \geq H_s^4\) yields a lower bound on the VEV of curvaton field, namely
\[
\sigma_\sigma \geq \frac{H_s^2}{m}. \tag{30}\]

Since \(m < H_s\), \(\sigma_\sigma > H_s\) which is the condition for the validity of the semi-classical description.

Secondly lets take into account the horizon temperature \(H_s\) of a quasi-de Sitter space. Since the curvaton mass is much smaller than \(H_s\), the curvaton “particles” can be taken as the relativistic particles. If the initial temperature of curvaton particles is lower than \(H_s\), the quantum fluctuations of the background will automatically heat their temperature up to \(H_s\). So the energy density of curvaton field should not be smaller than \(H_s^4\).

Here we also give the third argument. In curvaton model the curvaton mass is assumed to be much smaller than the Hubble parameter during inflation, which means the Compton wavelength is large compared to the curvature radius of the de Sitter space \(H^{-1}\). So the gravitational effects may play a crucial role on the behavior of the curvaton field. The well-known Bunch–Davies expression [30–32] for the average square value of a light scalar field \((m \ll H_s)\) in a quasi-de Sitter space takes the form
\[
\langle \sigma^2 \rangle = \frac{3H_s^4}{8\pi^3m^2}. \tag{31}\]
An intuitive understanding was given in [33]. According to the long-wave quantum fluctuation of a light scalar field \( m \ll H \) in inflationary universe, the behavior of such a light scalar field can be taken as a random walk [34]:

\[
\langle \sigma^2 \rangle = \frac{H^3}{4\pi^2} t.
\]

(32)

On the other hand, a massive scalar field cannot grow up to arbitrary large vacuum expectation value because it has a potential. The long wavelength modes of the light scalar field are in the slow-roll regime and obey the slow-roll equation of motion, i.e.

\[
3H \frac{d\sigma}{dt} = -\frac{dV(\sigma)}{d\sigma} = -m^2 \sigma.
\]

(33)

Combining these two considerations, in [33] Linde and Mukhanov proposed

\[
\frac{d\langle \sigma^2 \rangle}{dt} = \frac{H^3}{4\pi^2} - 2m^2 \langle \sigma^2 \rangle.
\]

(34)

We see that \( \langle \sigma^2 \rangle \) stabilizes at the point of \( \langle \sigma^2 \rangle = \frac{3H^4}{8\pi^2 m^2} \), which is the same as Eq. (31). This vacuum expectation value of curvaton mainly comes from the perturbation mode with wavelength \( \frac{m^2}{H^2} H^{-1} \) which stretched outside the horizon at the number of e-folds \( H^2 t/m^2 \). The inflation models with small total number of e-folds are artificial and the long stage of inflation is expected generically [35,36]. Since the wavelength is much larger than the Hubble horizon, this fluctuation mode is frozen to be a classical one and provides a non-zero classical configuration for curvaton field. The typical value of curvaton field in such a background is estimated as

\[
\sigma_* \sim \frac{H^2}{m}.
\]

(35)

Similar estimation is also adopted in the literatures, for example [37].

The value of \( \sigma_* \) is related to the Hubble parameter and \( f_{\text{NL}} \) through the WMAP normalization by Eq. (21). Eq. (35) reads

\[
m \frac{f_{\text{NL}}}{H_*} \sim 2.68 \times 10^3.
\]

(36)

Because the mass of curvaton is smaller than \( H_* \), the non-Gaussianity parameter is less than \( 2.68 \times 10^3 \).

The curvaton should decay before neutrino decoupling; otherwise, the curvaton perturbations may be accompanied by a significant isocurvature neutrino perturbation. The temperature of the universe at the moment of neutrino decoupling is roughly \( T_{\text{nd}} = 1 \) MeV. So the curvaton decay rate is bounded by the Hubble parameter at the moment of neutrino decoupling, namely

\[
\Gamma \geq \Gamma_0 = \frac{T_{\text{nd}}^2}{M_p} = 4.1 \times 10^{-25} \text{ GeV}.
\]

(37)

Considering (22) and \( m < H_* \), we find

\[
f_{\text{NL}} \lesssim \frac{6}{M_p^2 H_*^2} \frac{H_*}{T_{\text{nd}}} \left( \frac{H_*}{2.71 \times 10^7 \text{ GeV}} \right)^{\frac{5}{2}}.
\]

(38)

Similarly combining (22) and (36), we have

\[
f_{\text{NL}} \lesssim \left( \frac{m}{4.9 \times 10^4 \text{ GeV}} \right)^{\frac{5}{2}}.
\]

(39)

For a large non-Gaussianity \( f_{\text{NL}} = 10^2 \), the bounds on the Hubble parameter and the mass of curvaton become

\[
H_* \gtrsim 1.71 \times 10^6 \text{ GeV}, \quad m \gtrsim 1.23 \times 10^7 \text{ GeV}.
\]

(40)

The above constraints on the Hubble parameter and curvaton mass are not restricted.

In [38,39] the authors also suggested that the curvaton decay rate is at least of order \( Gm^3 \sim m^3/M_p^2 \) corresponding to gravitational-strength interactions. So we have

\[
\Gamma \gtrsim \Gamma_0 \approx \frac{m^3}{M_p^2}.
\]

(41)

Now Eq. (22) reads

\[
f_{\text{NL}} \lesssim 10^6 \frac{H_*^2}{m M_p}.
\]

(42)

Including (36), we find

\[
f_{\text{NL}} \lesssim \left( \frac{m}{3.4 \times 10^5 \text{ GeV}} \right)^{\frac{1}{2}}.
\]

(43)

If \( f_{\text{NL}} = 10^2 \) the curvaton mass should be larger than \( 3.4 \times 10^{11} \) GeV. Combing (36), (42) and (26), we find the typical values of \( f_{\text{NL}} \) and \( \tau_{\text{NL}} \) are bounded by the tensor–scalar ratio \( r \) from above:

\[
f_{\text{NL}} \lesssim 518 \cdot r^{\frac{1}{2}}.
\]

(44)

\[
\tau_{\text{NL}} \lesssim 3.86 \times 10^5 \cdot r^{\frac{1}{2}}.
\]

(45)

Or equivalently,

\[
H_* \gtrsim 9 \times 10^8 \cdot f_{\text{NL}} \text{ GeV}.
\]

(46)

Present bound on the tensor–scalar ratio is \( r < 0.20 \) and then \( f_{\text{NL}} \lesssim 346 \). If \( f_{\text{NL}} = 10^4 \), \( r \gtrsim 1.4 \times 10^{-3} \), \( H_* \gtrsim 9 \times 10^{12} \) GeV and inflation scale \( V^{1/4} \gtrsim 6.2 \times 10^{15} \) GeV which implies that inflation happened around GUT scale. The inequality (44) is saturated only when Eq. (41) are saturated. So the decay rate of curvaton cannot be much larger than gravitational-strength decay; otherwise the non-Gaussianity will be small.

The present and next generation of experiments, such as WMAP and PLANCK, will increase the accuracy to about \( 4 \cdot 10^{-4} \). We see that the non-Gaussianity parameter \( \eta_{\sigma\sigma} \) is bounded by the non-Gaussianity parameter \( f_{\text{NL}} \) from below, namely

\[
\eta_{\sigma\sigma} \sim \frac{m^2}{4H_*^4} \sim 4.64 \times 10^{-8} \cdot f_{\text{NL}}^2.
\]

(49)

In curvaton scenario the curvature perturbation is dominated by curvaton perturbation and then the slow-roll parameter \( \epsilon_H \) has a lower bound (25) which says

\[
\epsilon_H > \frac{r}{16} \gtrsim 8.68 \times 10^{-13} \cdot f_{\text{NL}}^2.
\]

(49)

If \( f_{\text{NL}} = 10^2 \), \( 2\eta_{\sigma\sigma} \sim 2 \times 10^{-4} \) and \( 2\epsilon_H \sim 1.74 \times 10^{-4} \). We see that the non-Gaussianity cannot give us a useful constraint on the spectral index.

4. Spectral index of the primordial power spectrum

Spectral index of the primordial power spectrum is another important quantity for characterizing the physics in the early Universe. WMAP3 [16] prefers a red tilted primordial power spectrum with \( n_s = 0.960 ^{+0.014}_{-0.013} \) for LCDM model, and \( n_s > 1 \) is disfavored even when gravitational waves \( r < 0.2 \) at 95% C.L. are included.

The spectral index in the curvaton scenario is given by

\[
n_s - 1 = 2\eta_{\sigma\sigma} - 2\epsilon_H = \frac{2m^2}{3H_*^4} - 2\epsilon_H.
\]

(47)

According to Eq. (36), we see that \( \eta_{\sigma\sigma} \) is bounded by the non-Gaussianity parameter \( f_{\text{NL}} \) from below, namely

\[
\eta_{\sigma\sigma} \sim \frac{m^2}{4H_*^4} \sim 4.64 \times 10^{-8} \cdot f_{\text{NL}}^2.
\]

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If \( f_{\text{NL}} = 10^2 \), \( 2\eta_{\sigma\sigma} \sim 2 \times 10^{-4} \) and \( 2\epsilon_H \sim 1.74 \times 10^{-4} \). We see that the non-Gaussianity cannot give us a useful constraint on the spectral index.
Usually we assume $m \ll H$, and then $\eta_\sigma \simeq 0$. To get the observed spectral index we need $\epsilon_H \simeq 0.02$ in curvaton scenario. Among so many inflation models, large-field models, such as chaotic inflation, give roughly the suitable value of $\epsilon_H$. Large field means the VEV of inflaton is larger than the Planck scale. However in [40,41] we argued that the VEV of a scalar field should be less than the Planck scale in a consistent low-energy effective field theory coupled to gravity. On the other hand, the Lyth bound [42] for single-field slow-roll inflation is roughly given by

$$\frac{|\Delta \phi|}{M_p} < \sqrt{2\epsilon_H \Delta N_e}, \quad (50)$$

where $N_e$ is the number of e-fold before the end of inflation. Requiring $|\Delta \phi|/M_p < 1$ yields

$$\epsilon_H < \frac{1}{2(\Delta N_e)^2}. \quad (51)$$

For $\Delta N_e = 50$, $\epsilon_H < 2 \times 10^{-4}$. So it is reasonable to expect that a spectral index indistinguishable from 1 is obtained in curvaton scenario. This is also pointed out in [43]. In inflation model (n, 1) is mainly due to a negative $\eta_\phi = N_p^2 \frac{d^2 V(\phi)}{d \phi^2}$. For the detail see [41].

A possible way to avoid the above bound is to consider multi-field inflation model. The simplest one is assisted inflation. Since there is a unique attractor behavior $\phi_1 = \phi_2 = \cdots = \phi_N$, the generalized Lyth bound reads

$$\frac{|\Delta \phi|}{\Lambda_G} = \sqrt{2\epsilon_H \Delta N_e}, \quad i = 1, 2, \ldots, N, \quad (52)$$

where $\Lambda_G = M_p/\sqrt{N}$ is the gravity scale for $N$ species [44,45]. In [44] we gave several examples to support that the variation of each inflaton should be less than $\Lambda_G$ in the assisted inflation model. If so, the multi-field inflation cannot help us to release the constraint on the slow-roll parameter $\epsilon_H$.

Another possibility is chain inflation in string landscape [46,47]. In this scenario, the universe tunneled rapidly through a series of metastable vacua with different vacuum energies. Since the total energy density is dominated by the vacuum energies, we get an inflationary universe. Chain inflation is not really a slow-roll inflation model and it does not suffer from the above constraints. In [47] we have $\epsilon_H = \frac{\eta_\sigma}{N_e}$. For $N_e = 50$, $\epsilon_H = 0.0067$, $\eta_\sigma = 0.987$. Many light scalar fields are expected to emerge in four-dimensional effective field theory from string theory. Some of them can be taken as curvatons. The bound on the non-Gaussianity parameter is $f_{NL} \lesssim 296$. Chain inflation might be generic in string landscape. So the experiment data can be nicely explained in string landscape.

Because the curvature perturbation is dominated by curvaton perturbation in curvaton scenario, we have $r < 16\epsilon_H$ and

$$f_{NL} \lesssim 871 \cdot \left( \frac{1}{\Delta N_e} \right)^2, \quad (53)$$

here we take (44) and (50) into account. If $|\Delta \phi|/M_p < 1$ for $\Delta N_e = 50$, $f_{NL} \lesssim 123$ which is roughly saturated the experiment bound.

5. Discussions

An unambiguous detection of $f_{NL} > 10$ will rule out most of the existing inflation models. The uncertainty of $f_{NL}^{local}$ from WMAP will shrink to be 42 for 8 years data, and 38 for 12 years data [15]. The accuracy of PLANCK will be roughly 6, if the local-type non-Gaussianity is of order 10$^{-5}$, it will be detected at high confidence level in the near future.

In this Letter we investigate the curvaton scenario in detail and find that the inflation scale is roughly at GUT scale in order to get a large local-type non-Gaussianity. Ekpyrotic model can also provide a large local-type non-Gaussianity. However unlike the slightly red-tipped gravitational wave spectrum in the inflation/curvaton model, the gravitational wave spectrum in Ekpyrotic model is correctly blue and then the amplitude is exponentially suppressed on all observable scales [48]. Gravitational wave perturbation can be used to distinguish curvaton scenario from Ekpyrotic model.

Here we also want to clarify two points in this Letter. One is that maybe we miss some order-one coefficients in (35) and (41), and then the bound in (44) should be modified to be a little looser or more stringent. But the order of magnitude of our result can be trusted. The other is that the bound in (44) depends on the WMAP normalization $P_{\Lambda WMAP} = 2.457 \times 10^{-5}$. If we let the normalization of the density perturbation free, we find

$$f_{NL} \lesssim \gamma^{-1} \cdot 518 \cdot r^4, \quad (54)$$

and Eq. (53) becomes

$$f_{NL} \lesssim \gamma^{-1} \cdot 871 \cdot \left( \frac{|\Delta \phi|}{\Delta N_e M_p} \right)^2, \quad (55)$$

where $\gamma = (P_{\gamma}/P_{\Lambda WMAP})^2$. Larger the amplitude of the density perturbation, smaller the non-Gaussianity. If we also consider $|\Delta \phi|/M_p < 1$ for $\Delta N_e = 50$, $f_{NL} \lesssim 123 \cdot \gamma^{-1}$. The density perturbation cannot be larger than 10 times of its observed value in our Universe if the non-Gaussianity parameter is not smaller than 10.

The equilateral-type non-Gaussianity has not been detected. Many models [49–53] were suggested to generate large equilateral-type non-Gaussianity as well. Other mechanisms concerning the large non-Gaussianity are discussed in [54–60]. Anyway, non-Gaussianity will be an important issue for cosmology and string theory.

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References
